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Fractional order PID: recent developments

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- Thanks go to Professors Liuping Wang and Antonio Visioli for organizing this wonderful PID Workshop @ IFAC-V WC 2020.
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Outline

I. Background and Research Questions

II. The Idea of "More Flat Phase" Design

III. FO-[PID] Controller Tuning

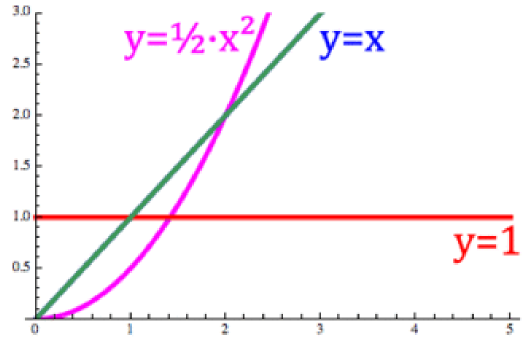
IV. The Animation of Feasible Regions of FOPI and IO-PID under Fairness Comparison Conditions

V. The FOPI application in compensation of actuator rate limit

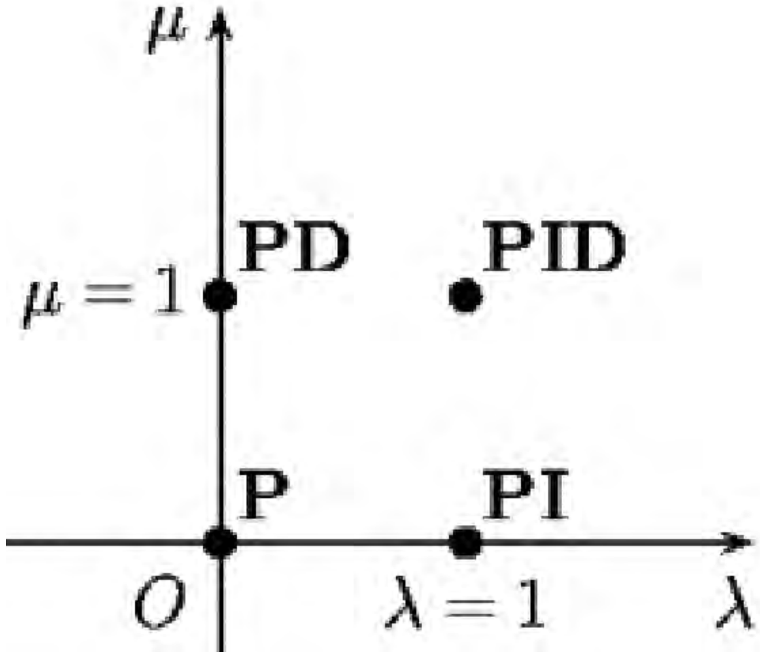
VI. Conclusions

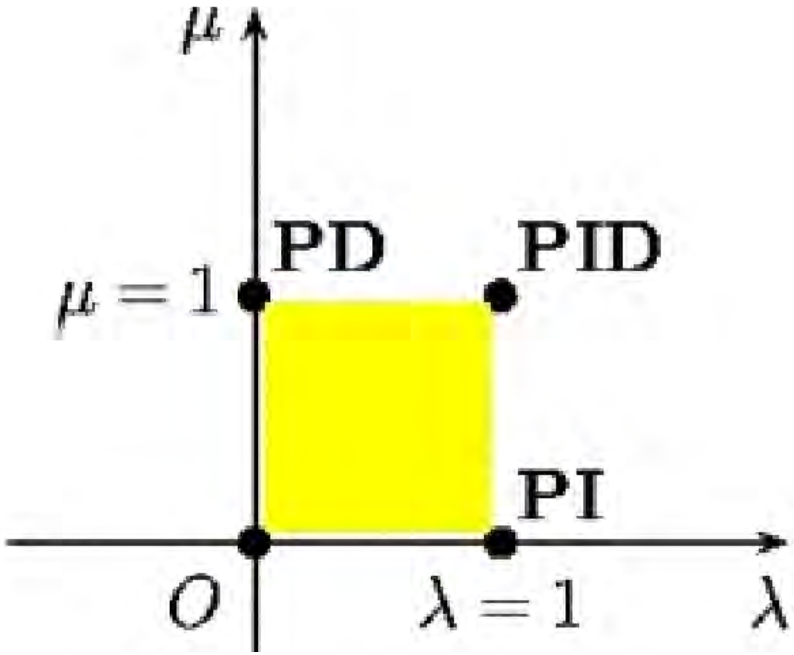
Fractional order PID control

- 90% are PI/PID type in industry. (**Ubiquitous**)



$$u(t)=K_p(e(t)+T_i\mathrm{D}_t^{-\lambda}e(t)+\frac{1}{T_d}\mathrm{D}_t^{\mu}e(t)).\qquad (\mathrm{D}_t^{(*)}\equiv_0\mathrm{D}_t^{(*)}).$$





Igor Podlubny. "Fractional-order systems and PI[l]D[μ]-controllers". IEEE Trans. Automatic Control,44(1): 208–214, 1999.
*YangQuan Chen, Dingyu Xue, and Huifang Dou. "**Fractional Calculus and Biomimetic Control**". IEEE Int. Conf. on Robotics and Biomimetics (RoBio04), August 22-25, 2004, Shenyang, China.*

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Control Engineering Practice

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Tuning and auto-tuning of fractional order controllers for industry applications

(Vol. 16, No. 7, pp. 798-812)

Christopher Greenwell, Publisher
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Control Engineering Practice

- Fractional Order System – official keyword of IFAC
- pid12.ing.unibs.it/

Background and Presentation Contents

- ✓ The fractional order controllers have attracted many attentions such as fractional order proportional integral derivative (FOPID) controller[1].
- ✓ The FOPID has five parameters to tune, which means the FOPID can offer better performance at the cost of extra implementation efforts than the regular PID.
- ✓ A tuning method with specification constraints, a specified gain crossover frequency, a specified phase margin and the flat phase constraint, is proposed to design the robust fractional order proportional integral (FOPI) controller[2], fractional order [proportional integral] (FO[PI]) controller[3] and fractional order [proportional derivative] (FO[PD]) controller[4].
- ✓ However, how to design a robust FOPI/FOPID controller systematically and theoretically still is an open question.

[1] Shah, P., and Agashe, S. (2016). Review of fractional PID controller. Mechatronics, volume (38), 29-41.
[2] Luo, Y., Chen, Y.Q., Wang, C.Y., et al. (2010). Tuning fractional order proportional integral controllers for fractional order systems. Journal of Process Control, volume (20), 823-831
[3] Luo, Y., and Chen, Y.Q. (2009). Fractional order [proportional derivative] controller for a class of fractional order systems. Automatica, volume (45), 2446-2450.
[4] Luo, Y., and Chen, Y.Q. (2012). Stabilizing and robust fractional order PI controller synthesis for first order plus time delay systems. Automatica, volume (48), 2159-2167.

Background and Presentation Contents

- How to design the satisfactory controller or what specifications should be satisfied ?

  For all controlled systems, the following specifications should be satisfied:

  ① A specified gain crossover frequency;

  ② A specified phase margin;

  ③ A flat phase constraint, which means that the open-loop phase is a constant around the given gain crossover frequency and can show the iso-damping property for the system response.

- How to evaluate the performance of one control strategy under fairness comparison conditions?

  (i) The performance indexes of the closed-loop system, such as ITAE, IAE and ISE;
  They are often used for the tuned controllers and different parameters result in different conclusions.

  (ii) The comparison of the feasibility regions with design specifications for different controllers.
  The feasibility regions offer a potential optimization space and a larger one means a larger possibility to find the optimal, unique controller.

Background and Presentation Contents

Based on the discussions above, the remaining questions are:

(1) How to design a robust fractional order PID controller systematically and theoretically when its parameter number is larger than three?

(2) Can the FOPID really outperform IOPID under fairness consideration?

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II. The Idea of "More Flat Phase" Design

III. FO-[PID] Controller Tuning

IV. The Animation of Feasible Regions of FOPI and IO-PID under Fairness Comparison Conditions

V. The FOPI application in compensation of actuator rate limit

VI. Conclusions

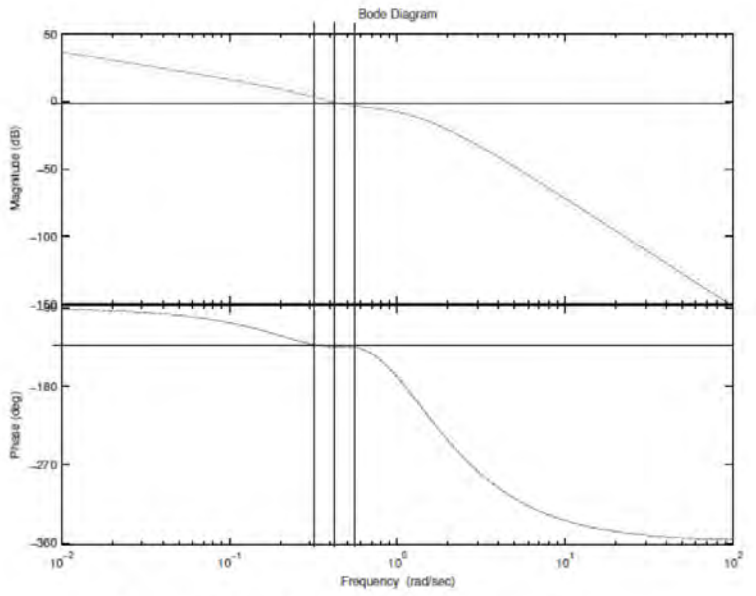
The Idea of "More Flat Phase" Design

Based on the discussions above, a robust fractional order controller should consider the following specifications :

① A specified gain crossover frequency, ω_{gc};

② A specified phase margin, ϕ_m;

③ A flat phase constraint, $\frac{d\varphi}{d\omega} = 0$.



(a) Basic idea: a flat phase curve at gain crossover frequency

However, when the parameter number of a robust fractional order PID controller is larger, how can we design a robust FOPID systematically and theoretically?

① A specified gain crossover frequency, ω_{gc};

② A specified phase margin, ϕ_m;

③ **More flat phase constraints**, $\begin{cases} \frac{d\varphi}{d\omega} = 0 \\ \cdots \\ \frac{d^{(n)}\varphi}{d\omega^{(n)}} = 0 \end{cases}$, n is the integer order > 1.

The Idea of "More Flat Phase" Design

The advantages of the idea of more flat phase design:

- ✓ More flat phase constraints means more possibilities that the open-loop phase is closer to constant and less sensitive to the gain variation for the closed-loop system.
- ✓ The number of constraints is equivalent to the parameter number of the FOPID controller, which results in a more reasonable parameter space.
- ✓ The FOPID controller designed with more flat phase constraints is more robust.

In the following section, the design method with more flat phase constraints is applied to FO[PID] controller to verify the superiority of the "more flat phase" idea.

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The FO-[PID] Controller Tuning

The control structure

- The control structure combining the controlled plant, the FO[PID] controller and the gain-phase margin tester[5] is shown in Fig. 1.
- $P(s) = \frac{K}{Ts+1} e^{-Ls}$ is first order plus time delay (FOPTD) systems.
- $C(s) = \left(K_p + \frac{K_i}{s} + K_d s\right)^r$ is the controller, namely, fractional order [Proportional Integral Derivative] (FO[PID]) controller, which has four parameters and $r \in (0,2)$.

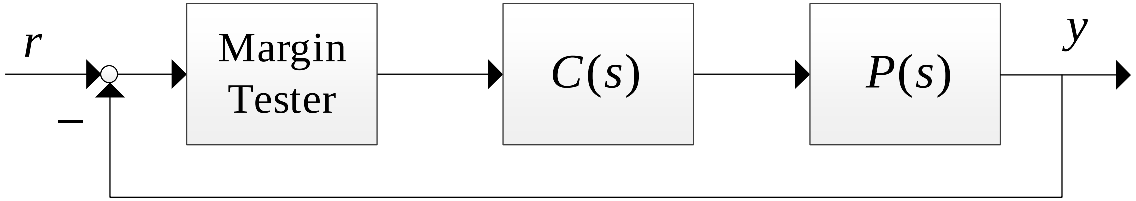

Fig. 1. The control structure with the margin tester.

- $M_T(A, \varphi) = Ae^{-j\varphi}$ is the gain-phase margin tester.
- We can obtain the parameter boundaries with the given gain margin (phase margin) when we set $\varphi = 0$ $(A = 1)$.

[5] Chang, C. H., and Han, K. W. (1990). Gain margins and phase margins for control systems with adjustable parameters. Journal of Guidance, Control, and Dynamics, volume (13), 404–408.

The FO-[PID] Controller Tuning

The stability region of the FO[PID] controller

- The characteristic equation of the closed-loop system in Fig. 1 can be depicted as,

$$D\left(K_d, K_p, K_i, r, A, \varphi; s\right) = (Ts+1)s^r + Ae^{-j\varphi}e^{-Ls}K\left(K_p s + K_i + K_d s^2\right)^r.$$

- The parameter boundaries of K_p and K_i with the fixed K_d and r can be determined by two parts, real root boundary (RRB) and complex root boundary (CRB).
- RRB: $K_i^r = 0$.
- CRB: $K_i = \begin{cases} \left|\sqrt{\frac{R^2}{1+(tan\theta)^2}}\right|, \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ -\left|\sqrt{\frac{R^2}{1+(tan\theta)^2}}\right|, \theta \in \left(-\pi, -\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \end{cases}, K_p = \begin{cases} \left|\frac{(K_i - K_d\omega^2)tan\theta}{\omega}\right|, \theta \in (0, \pi) \\ -\left|\frac{(K_i - K_d\omega^2)tan\theta}{\omega}\right|, \theta \in (-\pi, 0) \end{cases},$

where $R = \sqrt{\left(K_p\omega\right)^2 + (K_i - K_d\omega^2)^2}$ and $\theta = atan\frac{K_p\omega}{(K_i - K_d\omega^2)}$. With all different K_d and r, we can obtain the complete stability region of the FO[PID] parameters.

The FO-[PID] Controller Tuning

The FO[PID] controller design with the "more flat phase"

- A specified gain crossover frequency, ω_{gc}.
- A specified phase margin, setting $A = 1$ and $\varphi = \phi_m$.
- The more flat phase constraints,

$$\begin{cases} \dfrac{d\varphi}{d\omega} = \dfrac{rK_p(K_i+K_d\omega^2)}{(K_p\omega)^2+(K_i-K_d\omega^2)^2} - L - \dfrac{T}{1+(T\omega)^2} = 0 \\ \dfrac{d^2\varphi}{d\omega^2} = 2rK_p\dfrac{K_d\omega[(K_p\omega)^2+(K_i-K_d\omega^2)^2]}{[(K_p\omega)^2+(K_i-K_d\omega^2)^2]^2} - 2rK_p\dfrac{(K_i+K_d\omega^2)[K_p^2\omega-2K_d\omega(K_i-K_d\omega^2)]}{\left[(K_p\omega)^2+(K_i-K_d\omega^2)^2\right]^2} + \dfrac{2T^2\omega}{[1+(T\omega)^2]^2} = 0 \end{cases},$$

for the FO[PID] controller.

The FO-[PID] Controller Tuning

Tuning procedure

Step 1:

- Give a stable FOPTD system and two specifications, a specified phase margin, φ_m, and a specified gain crossover frequency, ω_{gc}.

Step 2:

- Fix K_d and r, and the CRB by setting $\varphi = \phi_m$ and $A = 1$ can be obtained in Fig. 2. Besides, the CRB can be obtained by sweeping over all K_d in Fig. 3.

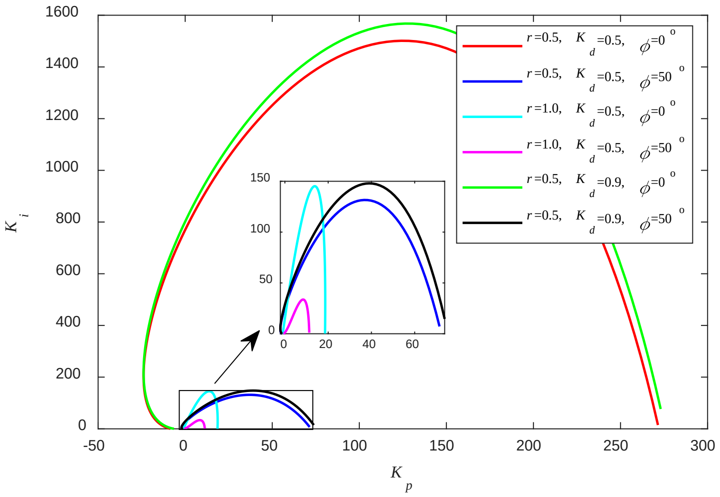

Fig. 2. The CRB with different K_d, φ_m and r.

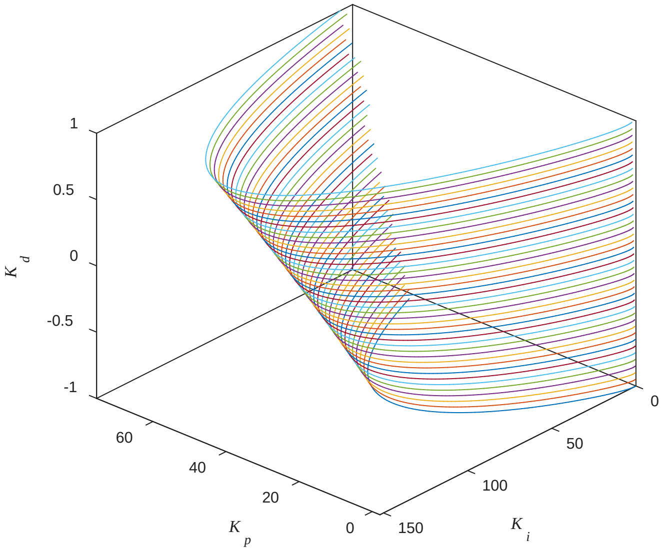

Fig. 3. The CRB by sweeping over K_d.

The FO-[PID] Controller Tuning

Tuning procedure

Step 3:

- Obtain one parameter band by satisfying ω_{gc}, φ_m and $\frac{d\varphi}{d\omega} = 0$ (red band in Fig. 4) and another parameter band by satisfying ω_{gc}, φ_m and $\frac{d^2\varphi}{d\omega^2} = 0$ (blue band in Fig. 4). Select any parameter pair in the intersection as the parameters of the FO[PID] controller.

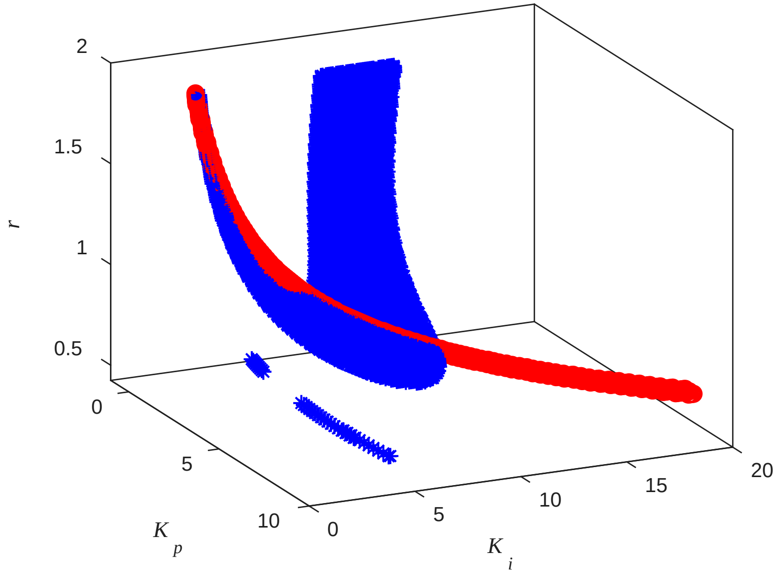

Fig. 4. The intersection of the two parameter bands.

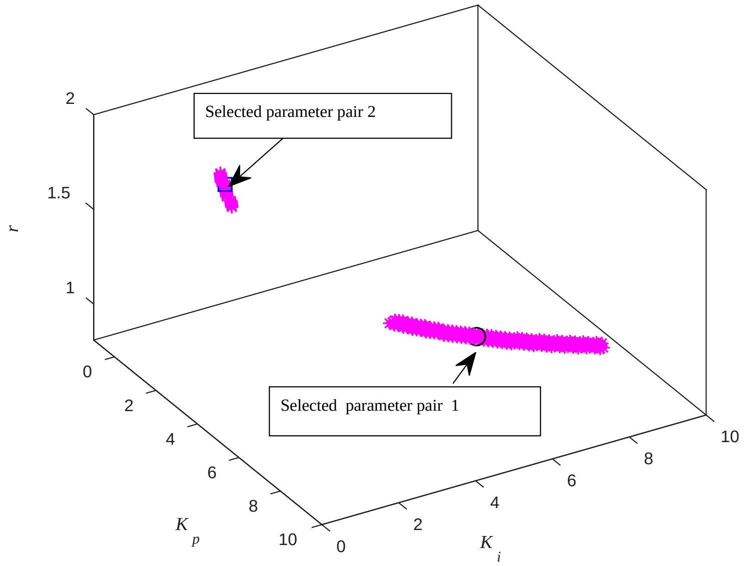

Fig. 5. The selected parameter pairs from the intersection

The FO-[PID] Controller Tuning

Tuning procedure

Remark 1:

- The frequency response and the time-domain response are presented in Fig. 6 and Fig. 7.
- We can know that the closed-loop system is not sensitive to the variation of the loop gain and the closed-loop system can obtain the satisfactory control performance with the proposed design method.

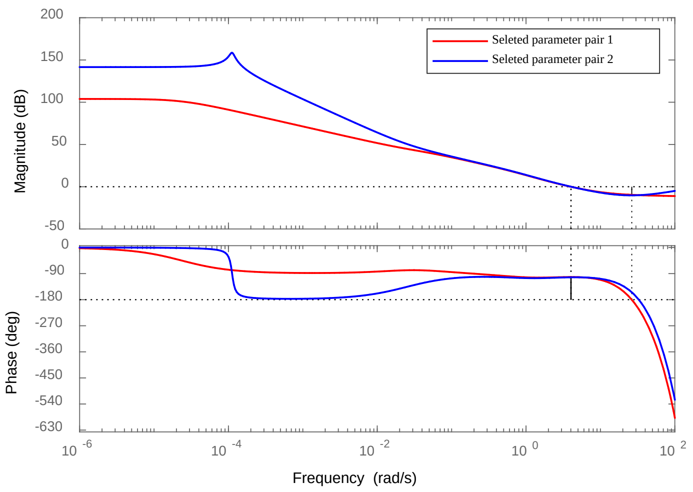

Fig. 6. The open-loop frequency response of the selected parameter pairs.

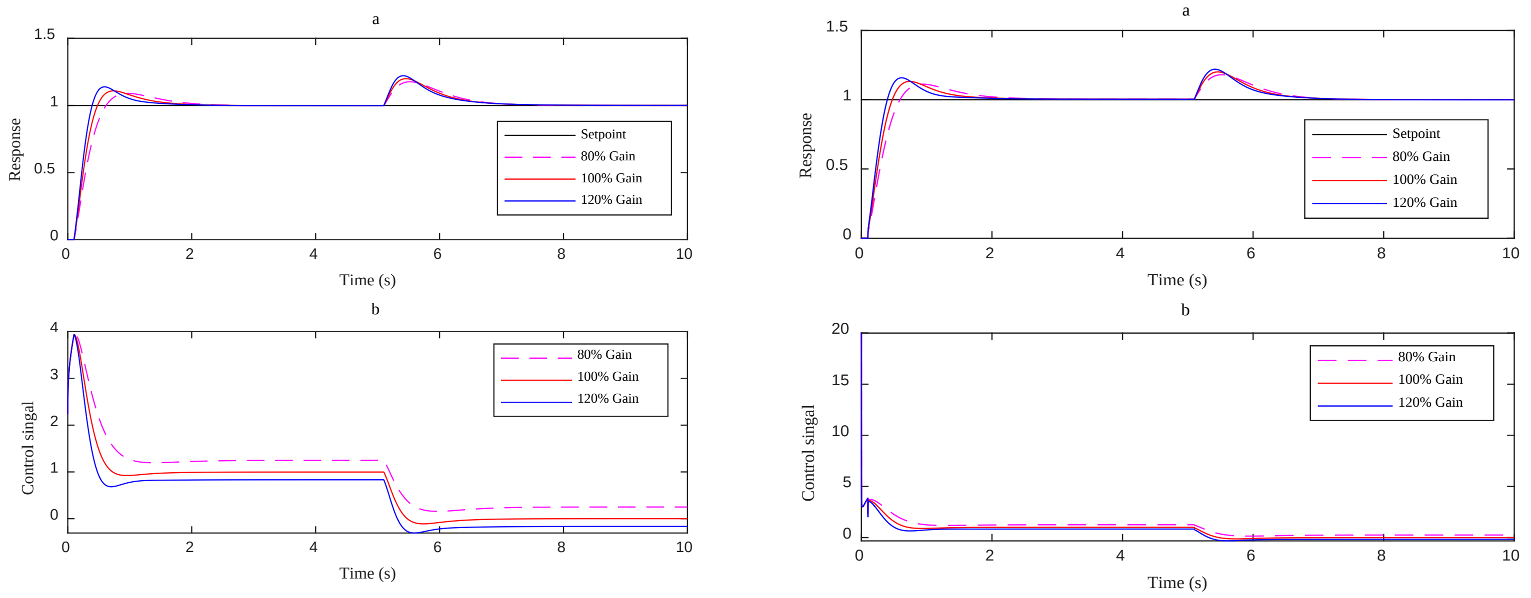

Fig. 7. The control performance with the selected parameter pairs.

The FO-[PID] Controller Tuning

Tuning procedure

Remark 2:

- With different specification constraints, ϕ_m and ω_{gc}, the control performance of FO[PID] and PID can be seen in Fig. 8 –Fig. 9.
- It can be learnt that the overshoot of PID has an obvious increase with the increasing gain.
- The proposed design method is not sensitive to the variation of the loop gain and can obtain the satisfactory control specifications.

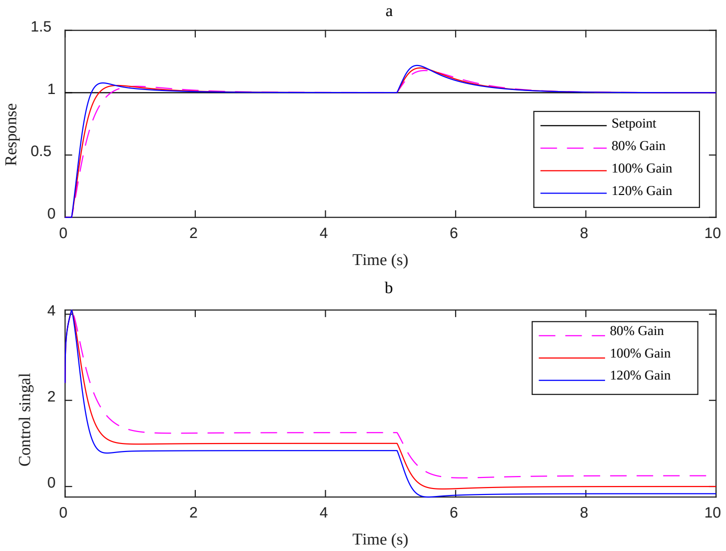

Fig. 8. The control performance of FO[PID] with different specification constraints.

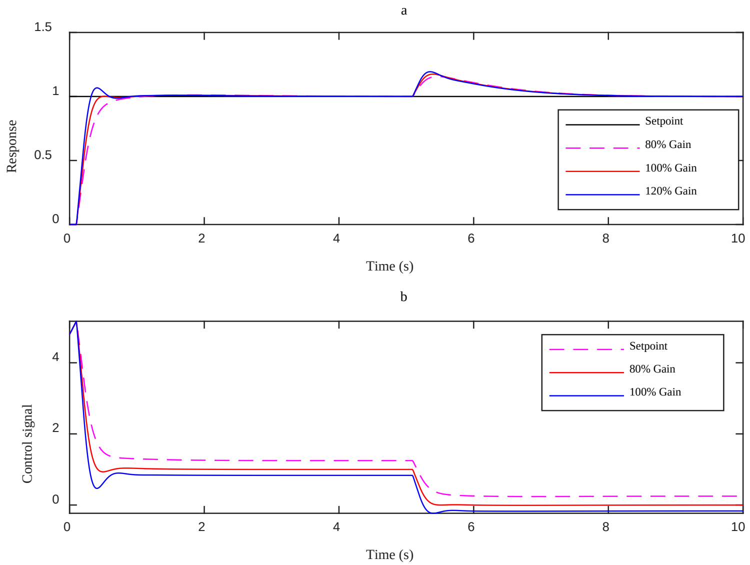

Fig. 9. The control performance of PID with the same specification constraints.

The FO-[PID] Controller Tuning

Tuning procedure

Step 4 (To obtain the achievable region of FO[PID]):

- By sweeping over all phase margin, $\phi_m \in (0, 180^0)$, and a specified gain crossover frequency, $\omega_{gc} \in (0, \omega_{max})$, the achievable region of FO[PID] can be obtained as in Fig. 10. Besides, the pseudo code can be seen in Fig. 11.

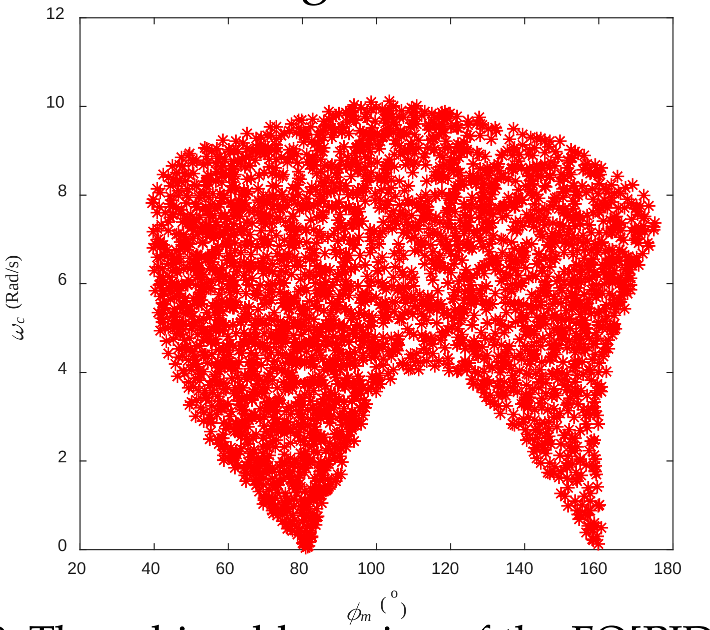

Fig. 10. The achievable region of the FO[PID] controller.

```
for ω=0: small interval : ω_max or a given ω
    for φ_m=0: small interval: π or a given φ_m
        for r=0.00001: small interval: 2
            for K_d=−T/AK: small interval: T/AK
                K_p = Eq.(14);
                K_i = Eq.(13);
                if Eq.(20) and Eq.(21) are satisfied for
the parameter pair {K_p, K_i}
                Mark the parameter pair {K_p, K_i, r,
K_d};
                Mark the corresponding pair {ω, φ_m};
                end
            end
end
end
end
plot {ω, φ_m}; % the achievable region
output {K_p, K_i, r, K_d}; % the parameter pair output
```

Fig. 11. The pseudo code of the design procedure.

The FO-[PID] Controller Tuning

Experimental verification

- The FO[PID] controller with the proposed design method is applied to the Peltier temperature control platform in Fig. 12. k_{gc} is added to the control signal to reflect the gain uncertainty.
- The closed-loop system with the designed FO[PID] controller is not sensitive to the variation of the loop gain and the time constant. The closed-loop system can obtain the satisfactory control performance as shown in Fig. 13.

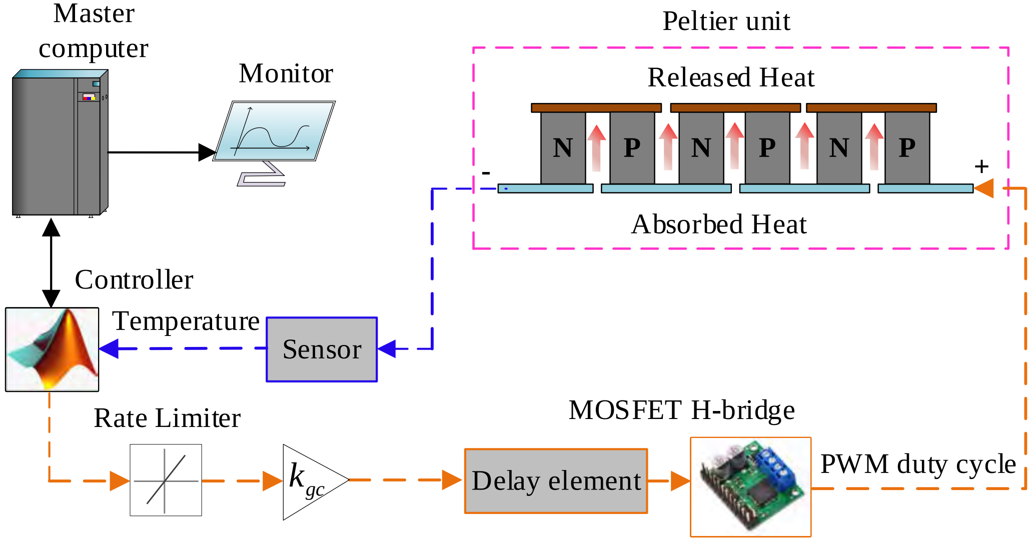

Fig. 12. The control structure of the Peltier platform.

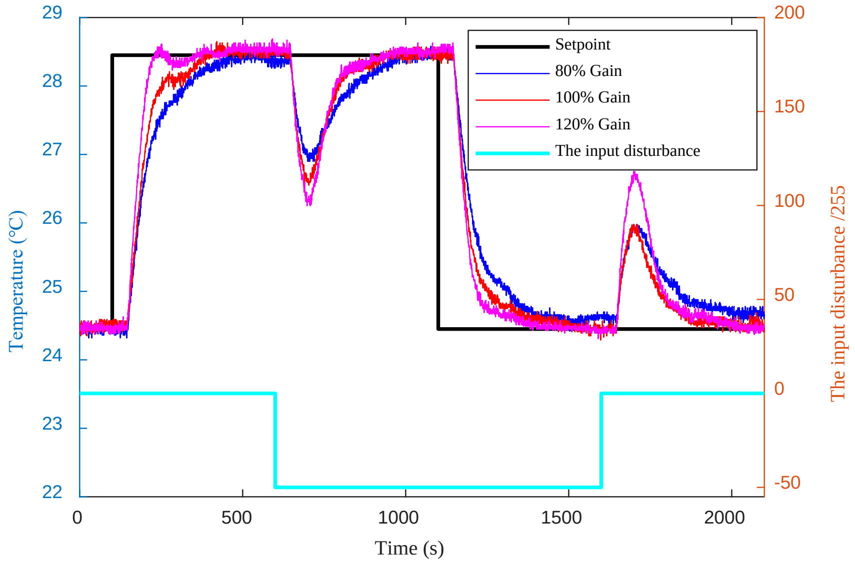

Fig. 13. The experiment result.

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The Animation of Feasible Regions of FOPI and IO-PID

Now, let's answer the second question: Can the FOPID really outperform IOPID under fairness comparison conditions?

Firstly, how to define the fairness comparison conditions?

Robustness *VS* Control Performance

The sizes of the feasibility regions of FOPI and IO-PID are the measurement with the constants of gain crossover frequency ω_{gc} and phase margin ϕ_m.

The Animation of Feasible Regions of FOPI and IO-PID

The stability region of the FOPI controller

- The characteristic equation of the closed-loop system in Fig. 1 ($C(s) = K_p + \frac{K_i}{s^r}$) can be depicted as,

$$D(K_d, K_p, K_i, r, A, \varphi; s) = (Ts+1)s^r + Ae^{-j\varphi}e^{-Ls}K(K_p s^r + K_i).$$

- The parameter boundaries of K_p and K_i with the fixed r can be determined by two parts, RRB and CRB.
- RRB: $K_i = 0$.
- CRB: $K_i = \frac{-(B_1S_1 + B_2C_1)}{AKS_2\omega^r}$, $K_p = \frac{B - (B_1S_1C_1 + B_2C_1^2)}{AKS_1} + \frac{B_1S_1C_2 + B_2C_1C_2}{AKS_2}$, (*)

where $B_1 = \omega^r C_2 - T\omega^{1+r}S_2$, $B_2 = \omega^r S_2 + T\omega^{1+r}C_2$, $C_1 = cos(\phi + \omega L)$, $C_2 = cos\frac{r\pi}{2}$, $S_1 = sin(\phi + \omega L)$, $S_2 = sin\frac{r\pi}{2}$, $E = K_i + K_p\omega^r C_2$, $F = K_p\omega^r S_2$. With all different r, we can obtain the complete stability region of the FOPI parameters.

The Animation of Feasible Regions of FOPI and IO-PID

The FOPI controller design with the specified constraints

- A specified gain crossover frequency, ω_{gc}
- A specified phase margin, setting $A = 1$ and $\varphi = \phi_m$.
- The flat phase constraint,

$$\frac{d\varphi}{d\omega} = \frac{\left(B_1^2 + B_2^2\right)(EF' - FE') + (B_1' B_2 - B_2' B_1)(EF' - FE')}{(B_1 E + B_2 F)^2 + (B_1 F - B_2 E)^2} - L = 0$$

where $E' = C_2 K_p r \omega^{r-1}$, $F' = S_2 K_p r \omega^{r-1}$, $B_1' = C_2 r \omega^{r-1} - S_2 T(1+r)\omega^r$, and $B_2' = S_2 r \omega^{r-1} + C_2 T(1+r)\omega^r$.

The Animation of Feasible Regions of FOPI and IO-PID

The design procedure or pseudo code to obtain feasibility regions

$for\ \ \omega_c = 0: small\ interval: \omega_{cmax}$

$for\ \phi_m = 0: small\ interval: \pi$

$for\ r = 0.0001: small\ interval: 2$

$K_p = Eq.(*.1);$

$K_i = Eq.(*.2);$

$if\ \ Eq.(10)\ is\ satisfied\ for$

$the\ pair\ \{K_p, K_i, r\}$

$Mark\ \ the\ parameter\ pair\ \{K_p, K_i, r\};$

$Mark\ \ the\ corresponding\ pair\ \{\omega_c, \phi_m\}.$

end

end

end

end

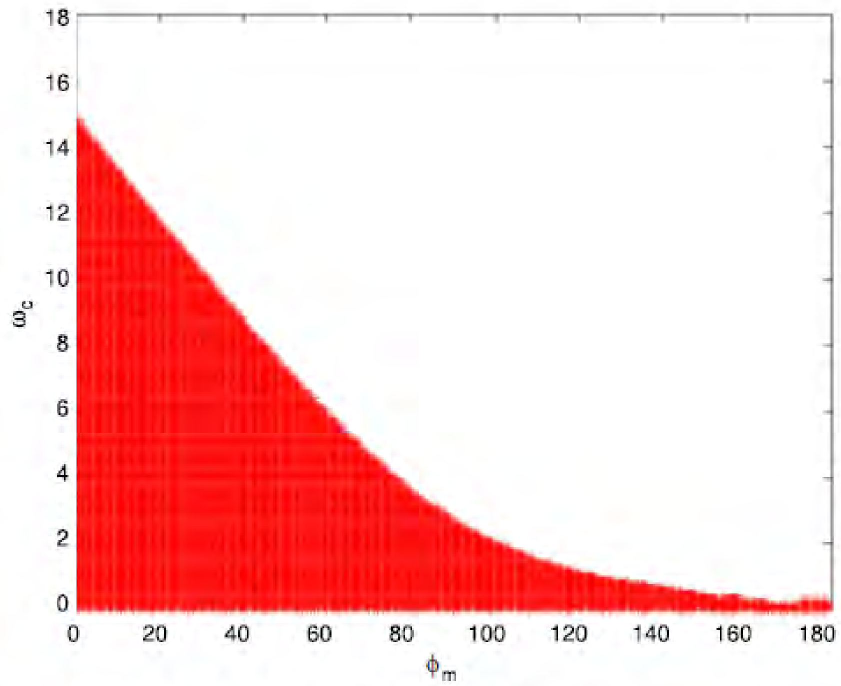
- $Plot\ \{\omega_c, \phi_m\}$ % (the feasibility regions)

***Remark* 1**: The code can be simplified for obtaining the parameter pair to satisfy the constants: the specific pair $\{\omega_c, \phi_m\}$ and the flat phase.

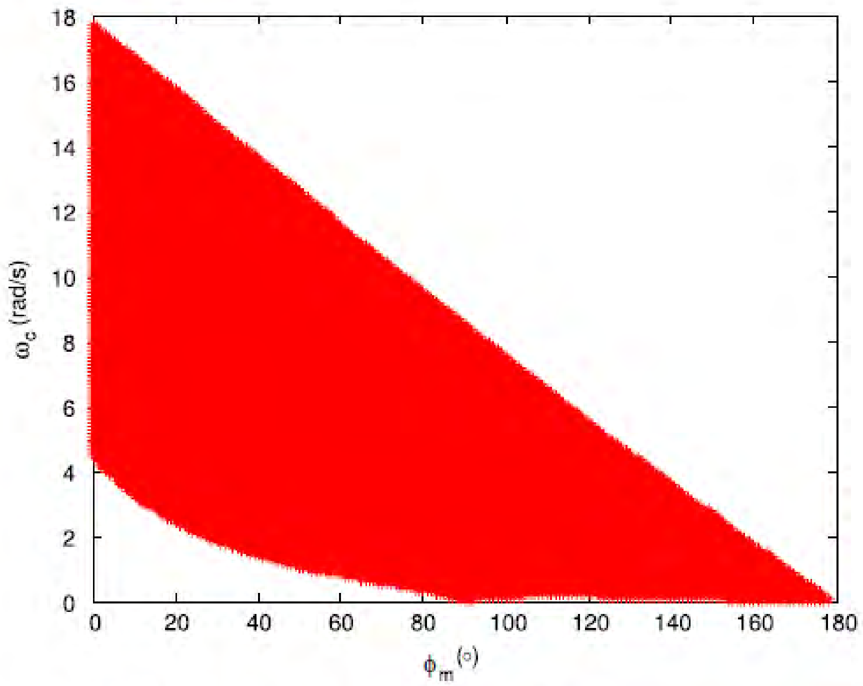
***Remark* 2**: Note that the feasibility regions of IOPID can be obtained with similar design procedure.

The Animation of Feasible Regions of FOPI and IO-PID

The feasibility regions with different delay times (1)



(a) FOPI with $T = 1$ s and $L = 0.1$ s.



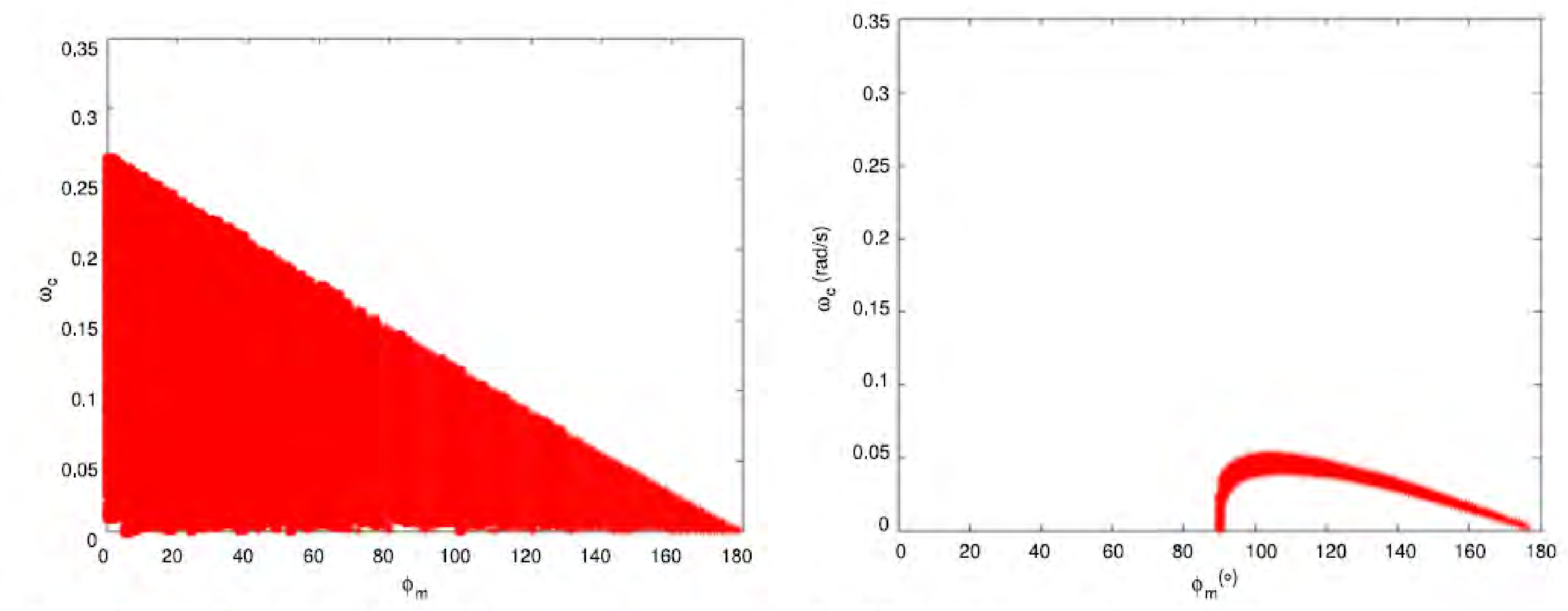
(b) IOPID with $T = 1$ s and $L = 0.1$ s.

Fig. 13. The feasibility regions of ω_c and ϕ_m for FOPI and IOPID design with $T = 1\,s$ and $L = 0.1\,s$.

The feasibility region of FOPI is smaller than that of IOPID when L is small. However, the feasibility set for FOPI covers the blank part of the feasibility set for IOPID with small values of ω_c and ϕ_m.

The Animation of Feasible Regions of FOPI and IO-PID

The feasibility regions with different delay times (2)



(a) FOPI with $T = 1$ s and $L = 10$ s.

(b) IOPID with $T = 1$ s and $L = 10$ s.

Fig. 14. The feasibility regions of ω_c and ϕ_m for FOPI and IOPID design with $T = 1\ s$ and $L = 10\ s$.

It can be seen clearly that, for the FOPI controller, the feasibility region of ω_c and ϕ_m is much bigger than that of the IOPID controller with the time delay is large.

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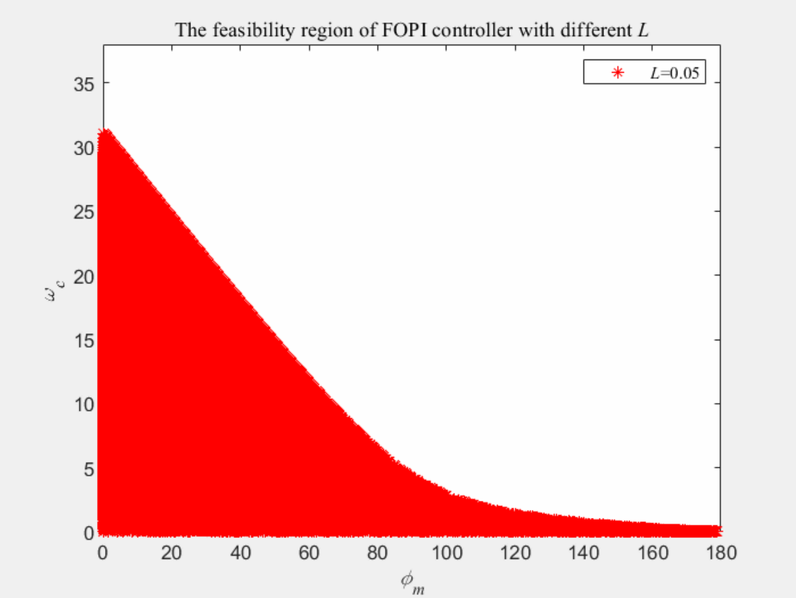
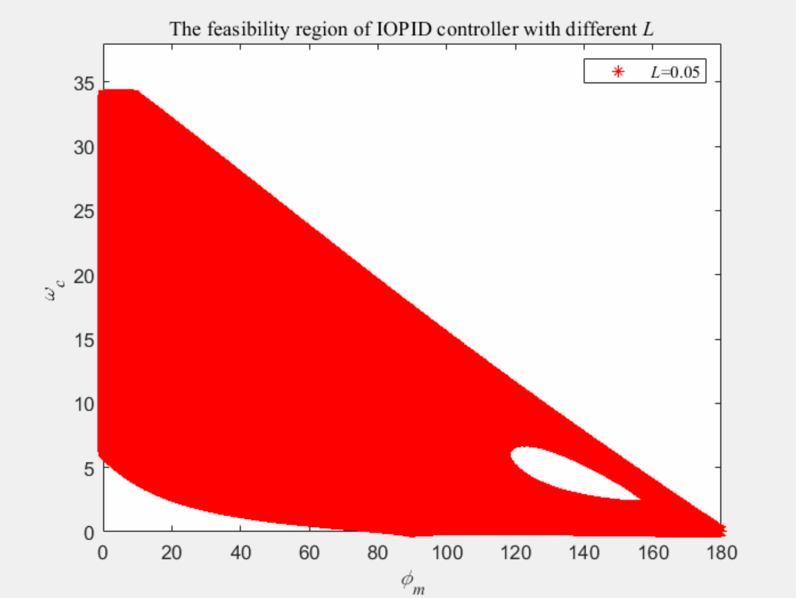
The Animation of Feasible Regions of FOPI and IO-PID

The feasibility regions with different delay times (3)

How the feasibility regions change when the time delay L is changing?
Animated feasibility regions can do!



(a) The FOPI with different L.



(b) The IOPID with different L.

Fig. 15. The feasibility regions of ω_c and ϕ_m for FOPI and IOPID design with $T \in [0.05,10]$.

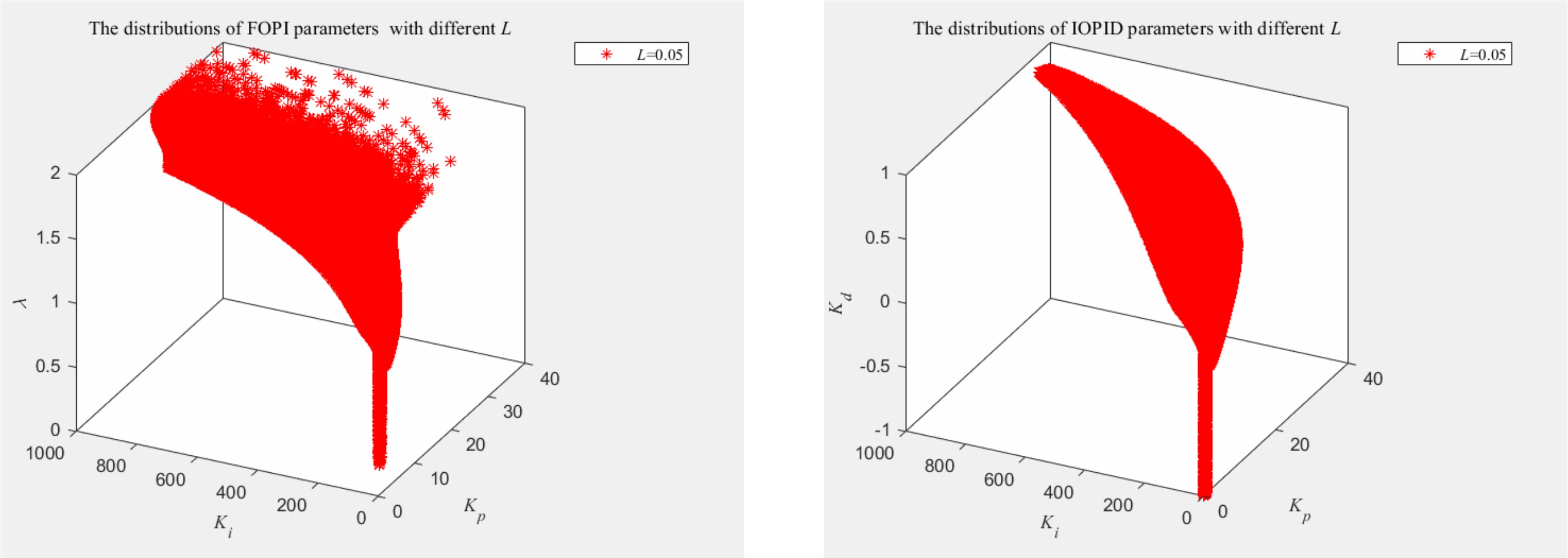
In summary:
When $L/(L+T) \to 1$, FOPI is more needed than IOPID.
When $L/(L+T) \to 0$, FOPI is the extended option of IOPID.

The FOPID really outperform IOPID under the fairness comparison conditions.

The Animation of Feasible Regions of FOPI and IO-PID

The feasibility regions with different delay times (4)

The distributions of FOPI and IOPID parameters with $L \in [0.05,10]$.



(a) The FOPI with different L.

(b) The IOPID with different L.

Fig. 16. The distributions of FOPI and IOPID parameters with $T \in [0.05,10]$.

It can be seen that, $\{K_p, K_i\}$ of FOPI in x −axis and y −axis is larger than that of IOPID.

The Animation of Feasible Regions of FOPI and IO-PID

The stability regions of FO[PI] controller

- RRB: $K_i^r = 0$.
- CRB: $K_i = \begin{cases} \left|\sqrt{\frac{R^2}{1+(tan\theta)^2}}\right|, \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ -\left|\sqrt{\frac{R^2}{1+(tan\theta)^2}}\right|, \theta \in \left(-\pi, -\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \end{cases}, K_p = \begin{cases} \left|\frac{(K_i - K_d\omega^2)tan\theta}{\omega}\right|, \theta \in (0, \pi) \\ -\left|\frac{(K_i - K_d\omega^2)tan\theta}{\omega}\right|, \theta \in (-\pi, 0) \end{cases},$

where $R = \sqrt{(K_p\omega)^2 + K_i^2}$, $\theta = atan\frac{K_p\omega}{K_i} \in (-\pi, \pi)$. With all different K_d and r , we can obtain the complete stability region of the FO[PI] parameters.

The FO[PI] controller design with the specified constraints

- A specified gain crossover frequency, ω_{gc}
- A specified phase margin, setting $A = 1$ and $\varphi = \phi_m$.
- The flat phase constraint,

$$\frac{d\varphi}{d\omega} = \frac{rK_pK_i}{(K_p\omega)^2 + K_i^2} - L - \frac{T}{1+(T\omega)^2} = 0.$$

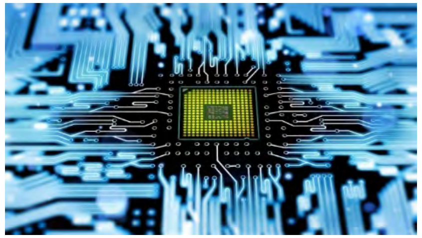
Outline

The FOPI application in compensation of actuator rate limit

Actuator rate limit

- Ignored in classical industry
  - ➢ Low requirements of control performance
  - ➢ Cost reduction

- Control challenge in high technology manufacturing
  - ➢ Precision control
  - ➢ Fast response
  - ➢ Robustness



The FOPI application in compensation of actuator rate limit

- What is actuator rate limit (rate limiter)?

✓ Input: $r(t) = Asin(\omega t)$ Output: $|\dot{x}(t)| \le R$ (R: rate limit value)

✓ Fully activated rate limiter: pure triangular output (dramatic magnitude reduction & phase delay)

- Rate limit effect: system identification & control performance

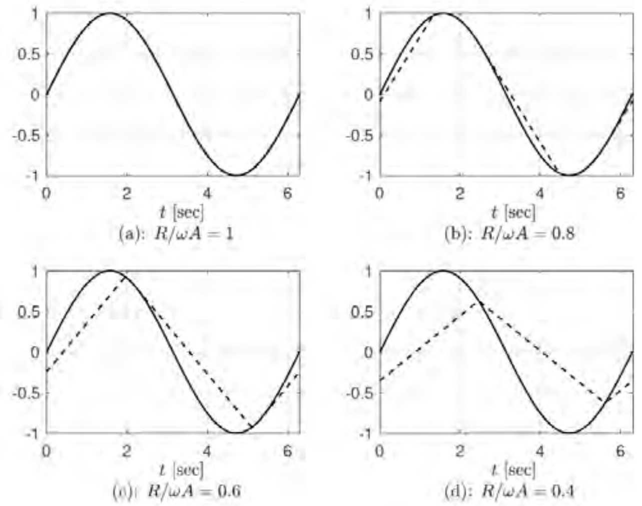


Fig. 17. Steady output (solid) of rate limiter under sinusoidal input (dashed).

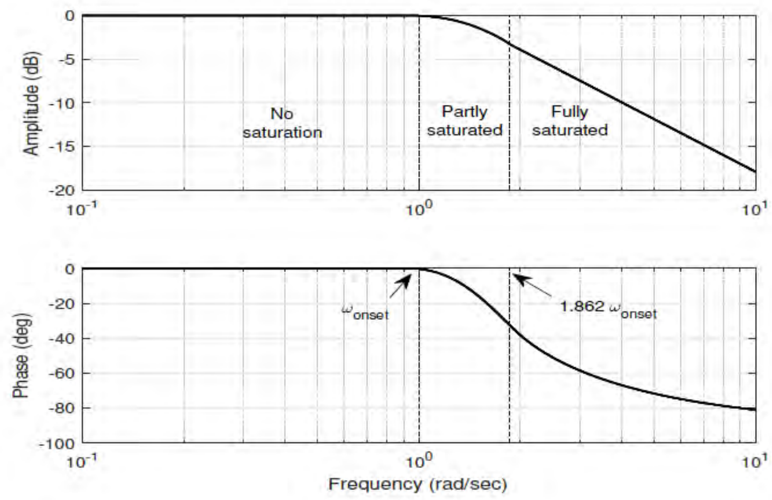


Fig. 18. Bode plot of rate limiter.

The FOPI application in compensation of actuator rate limit

- Rate limit effect on (step response based) system identification
- ✓ Step signal: infinite change rate (first derivative) at step time
- ✓ Smaller rate limit value generates more sluggish step response
- ✓ Traditional identification (without considering rate limit): mismatched model & unsatisfied control performance

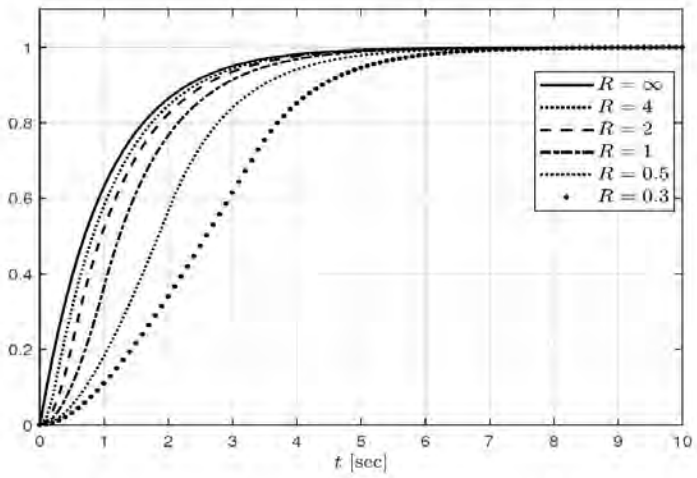


Fig. 19. Unit step response of $1/(s+1)$ under different rate limit value.

Real plant:

$$G(s) = \frac{K}{Ts+1} \quad \& \quad R$$

FOPTD model

$$\hat{G}(s) = \frac{\hat{K}}{\hat{T}s+1} e^{-\hat{L}s}$$

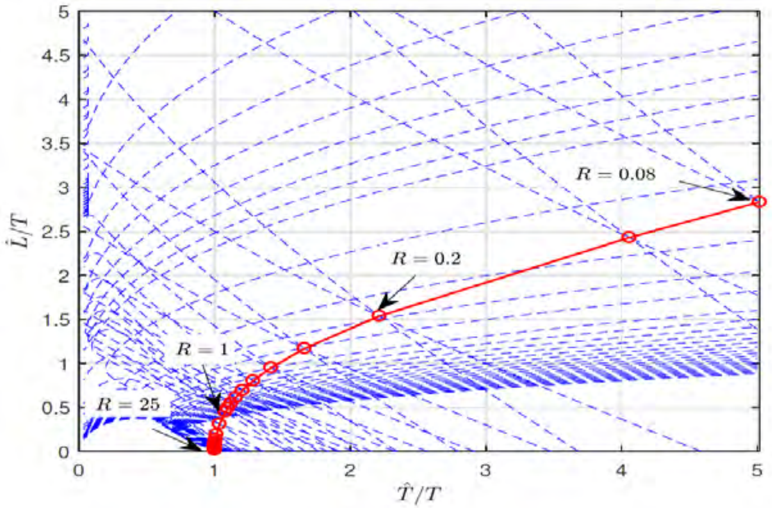


Fig. 20. Model mismatch of system $G(s)$ under rate limit.

The FOPI application in compensation of actuator rate limit

- Rate limit effect on control performance

✓ Real plant

$$G_0(s) = \frac{1}{s+1} \quad \& \quad R \qquad \text{(KLTR model)}$$

✓ Identified model (without rate limit)

$$\hat{G}(s) = \frac{\hat{K}}{\hat{T}s+1} e^{-\hat{L}s} \qquad \text{(KLT model)}$$

✓ Controller: optimal PI controller based on KLTR & KLT model (ISE index)

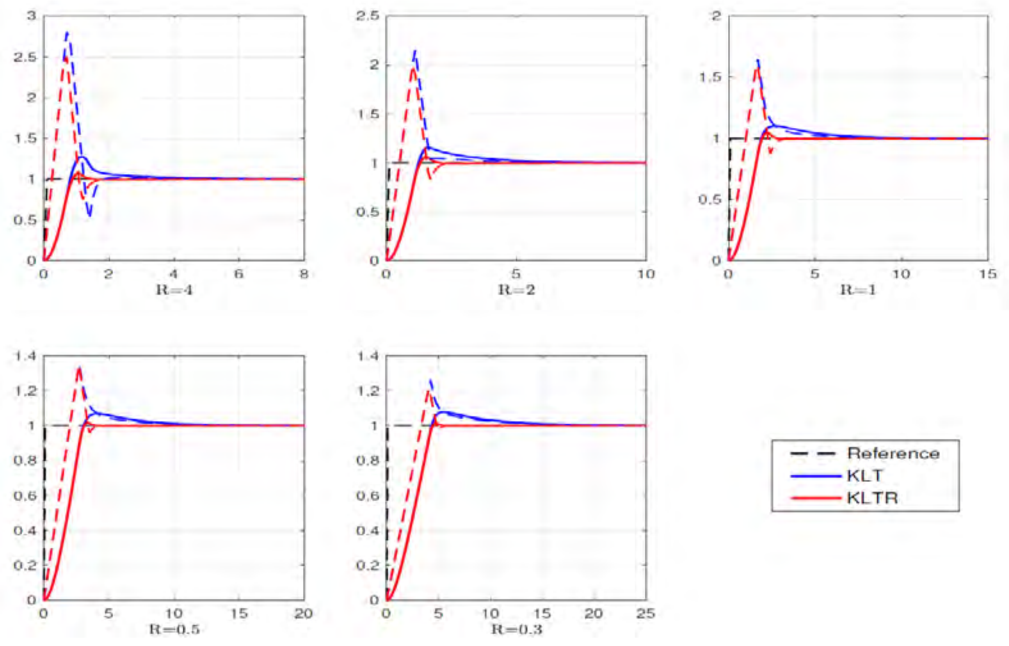


Fig. 21. Step response (solid) and control signal (dashed) of $G_0(s)$.

The FOPI application in compensation of actuator rate limit

- Control purpose: compensate phase delay generated by the actuator rate limit
- Controller: flat phase FOPI vs IOPID
- Compensation strategy:

✓ design initial controller
✓ analyze rate limit effect
✓ update design specifications
✓ redesign the controller

The FOPI application in compensation of actuator rate limit

Step 1: initial flat phase controller design

- Flat phase implementation

✓ Bode plot: $$\left.\frac{\mathrm{d}\big(\arg(C(j\omega)G(j\omega))\big)}{\mathrm{d}\omega}\right|_{\omega=\omega_b}=0$$

✓ Nyquist plot: $$\left.\arg\left(\frac{\mathrm{d}(C(j\omega)G(j\omega))}{\mathrm{d}\omega}\right)\right|_{\omega=\omega_b}=\Phi_m$$

- Flat phase in Nyquist plot can applied to general minimum-phase system

✓ $$\frac{\mathrm{d}L(j\omega)}{\mathrm{d}\omega}=G(j\omega)\frac{\mathrm{d}C(j\omega)}{\mathrm{d}\omega}+C(j\omega)G(j\omega)\left(\boxed{\frac{\mathrm{d}\ln|G(j\omega)|}{\mathrm{d}\omega}}+\boxed{\frac{\mathrm{d}\angle G(j\omega)}{\mathrm{d}\omega}}\right)$$

The FOPI application in compensation of actuator rate limit

Step 1: initial flat phase controller design

- Bode integrals (minimum-phase system):

✓ $\angle G(j\omega_0) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\mathrm{d}\ln|G(j\omega)|}{\mathrm{d}v} \ln\coth\frac{|v|}{2} \qquad v = \ln\frac{\omega}{\omega_0}$

✓ $\ln|G(j\omega_0)| = \ln|K| - \frac{\omega_0}{\pi}\int_{-\infty}^{\infty} \frac{\mathrm{d}(\frac{\angle G(j\omega)}{\omega})}{\mathrm{d}v} \ln\coth\frac{|v|}{2}$ (K: steady-state gain)

- Flat phase approximation

✓ $\left.\frac{\mathrm{d}\ln|G(j\omega)|}{\mathrm{d}\omega}\right|_{\omega_0} \approx \frac{2}{\pi\omega_0}\angle G(j\omega_0)$ or $\left.\frac{\mathrm{d}\ln|G(j\omega)|}{\mathrm{d}\omega}\right|_{\omega_0} \approx \frac{2}{\pi\omega_0}(\angle G(j\omega_0) + \tau\omega_0)$ $(G(s) = G_0(s)e^{-\tau s})$

✓ $\frac{\mathrm{d}\angle G(j\omega)}{\mathrm{d}\omega} \approx \frac{\angle G(j\omega_0)}{\omega_0} - \frac{2}{\pi\omega_0}(\mathrm{d}\ln|G(j\omega_0)| - \ln|K|)$

The FOPI application in compensation of actuator rate limit

Step 2: analyze rate limit effect in closed-loop system

- Describing function of the rate limiter

✓ $N(j\omega, \omega_{onset}) = \frac{4}{\pi}\frac{\omega_{onset}}{\omega} e^{-j\arccos\frac{\pi}{2}\frac{\omega_{onset}}{\omega}}$

Step 3: design specifications update rule

- Original: ω_b & Φ_m
- Update rule:

✓ $\omega_b^* = \omega_b |N(j\omega_b)|^{\frac{\pi\omega_b \ln 10}{2(\Phi_m - \pi)}}$ or $\omega_b^* = \omega_b |N(j\omega_b)|^{\frac{\pi\omega_b \ln 10}{2(\Phi_m - \pi + \tau\omega_b)}}$

✓ $\Phi_m^* \approx \Phi_m - \angle N\left(j\omega_b^*\right) - (G(j\omega_b) - \angle G(j\omega_b^*))$

Step 4: redesign the controller

The FOPI application in compensation of actuator rate limit

Simulation verification

- Plant model

✓ $G(s) = \frac{1}{2s+1}e^{-0.1s}$ $R = 1$

- Design specifications:

$\omega_b = 0.85\, rad/s$ & $\Phi_m = 50°$

- Initial flat phase controller

✓ $C_1(s) = 0.84 + \frac{2.17}{s} + 0.86s$

✓ $C_2(s) = 0.55 + \frac{1.57}{s^{0.91}}$

- Compensated controller

✓ $\tilde{C}_1(s) = 1.92 + \frac{1.64}{s} + 0.31s$

✓ $\tilde{C}_2(s) = 1.52 + \frac{1.43}{s^{0.90}}$

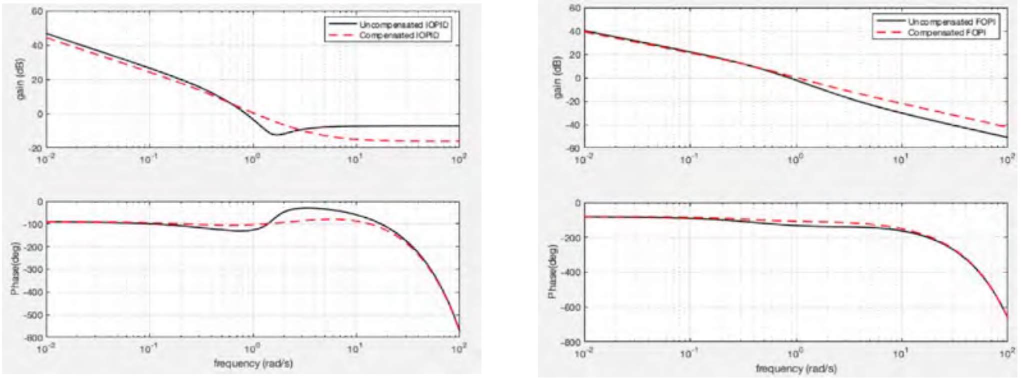


Fig. 22. Bode plots of open-loop system after compensation: IOPID(left); FOPI(right)

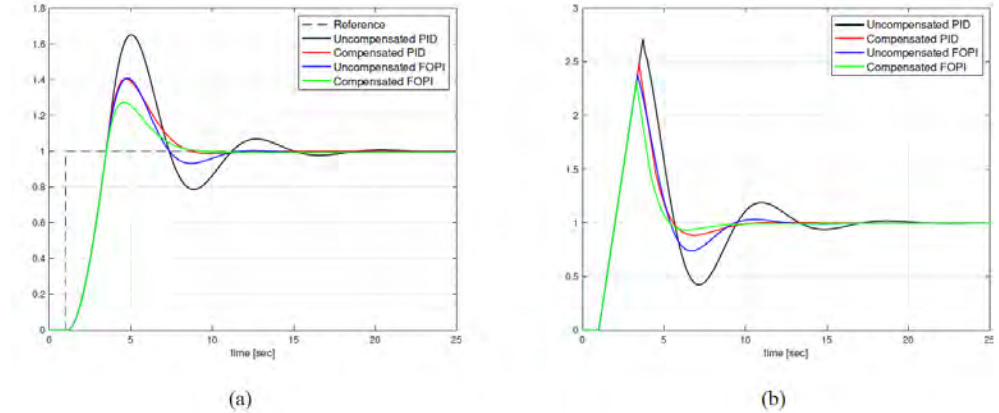


Fig. 23. Step response (a) and control signal.

The FOPI application in compensation of actuator rate limit

Experimental verification: Peltier temperature control platform

- Plant model

✓ $G(s) = \frac{0.1646}{51.5028s+1} e^{-1s} \quad R = 1.5\&1$

- Design specifications:

$\omega_b = 0.05\, rad/s \quad \& \quad \Phi_m = 45°$

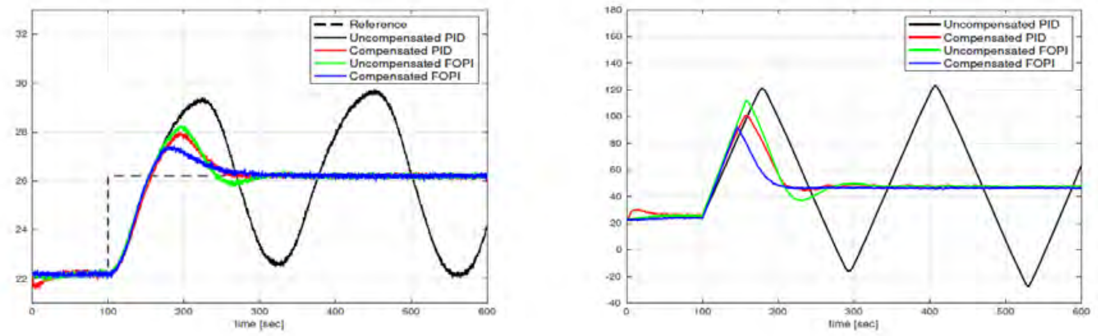
- Initial flat phase controller

✓ $C_1(s) = 7.53 + \frac{0.94}{s} + 75.86s \quad \& \quad C_2(s) = 6.15 + \frac{0.90}{s^{0.94}}$

- Compensated controller

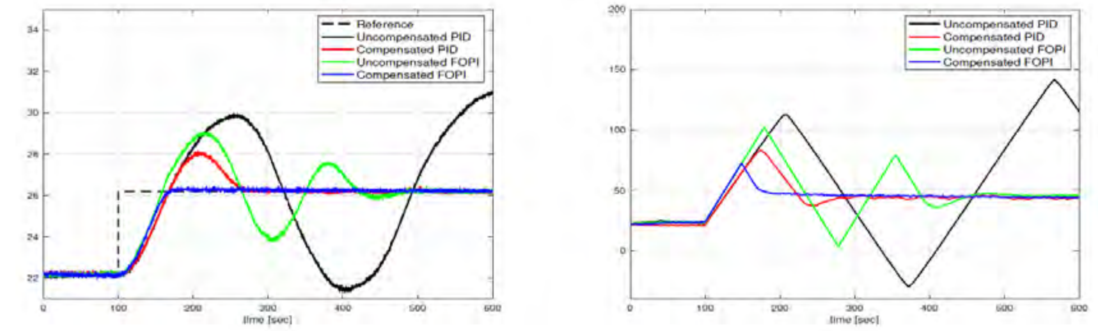
✓ $R = 1.5\text{:} \quad \tilde{C}_1(s) = 14.16 + \frac{0.56}{s} + 27.53s \quad \& \, \tilde{C}_2(s) = 12.85 + \frac{0.63}{s^{0.92}}$

✓ $R = 1\text{:} \quad \tilde{C}_1(s) = 16.46 + \frac{0.42}{s} + 53.76s \quad \& \, \tilde{C}_2(s) = 12.31 + \frac{1.21}{s^{0.58}}$



(a) (b)

Fig. 22. Step response (a) and control signal (R=1.5).



(a) (b)

Fig. 23. Step response (a) and control signal (R=1).

Conclusions

- The idea of the “more flat phase” is proposed to design the fractional order controller.
- The tuning procedure of the FO[PID] with “more flat phase” specification constraints is deduced and discussed.
- The effectiveness of the FO[PID] is verified by experiments.
- The FOPID really outperform IOPID under the fairness comparison conditions, which is verified by the animation of feasible regions of FOPI and IO-PID.
- The FOPI has been successfully applied to the compensation of actuator rate limit effect.

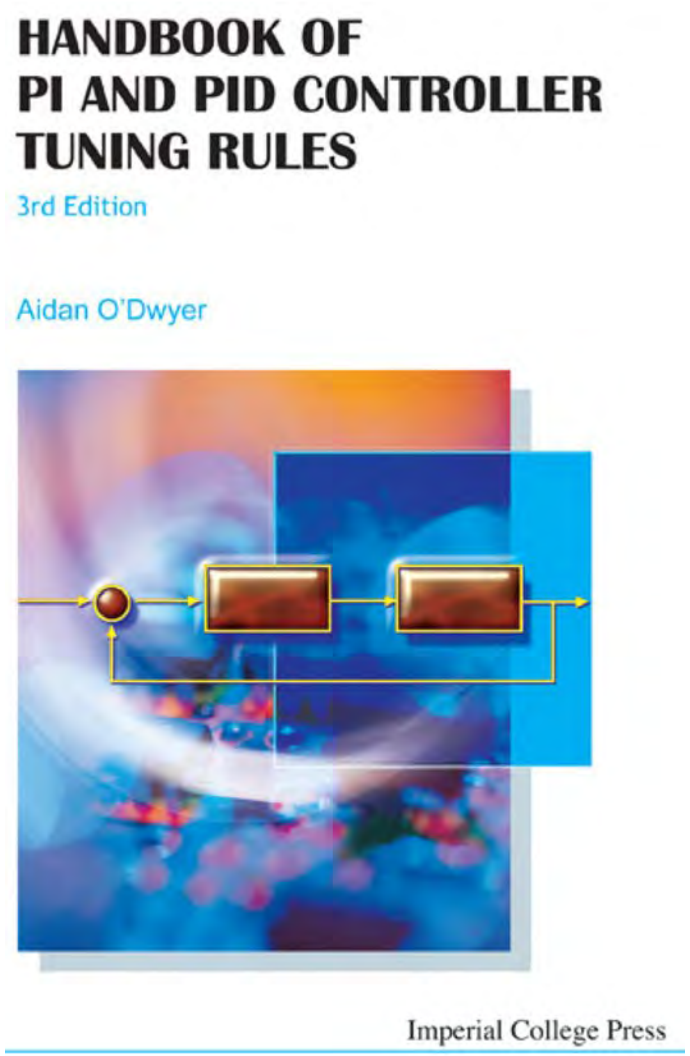
What's next?

- Fractional Order PID tuning,
- Smarter PID (digital twin, edge computing, embedded AI etc.)

Can PID still be PHD topic?

- Yes. Starting from this slide!

[1] IFAC PID 2018 Conference Plenary talk: "Fractional order PID control: better than the best issue and what's next" https://youtu.be/B3BurjUYPOA
[2] AA Dastjerdi, BM Vinagre, YQ Chen, SH HosseinNia. "Linear fractional order controllers; A survey in the frequency domain". Annual Reviews in Control, https://doi.org/10.1016/j.arcontrol.2019.03.8(49), 51-70, 2019
[3] Shah, P., and Agashe, S. (2016). Review of fractional PID controller. **Mechatronics**, volume (38), 29-41.



- 3^{rd} ed, 1935-2008, 600+ pages, 2009.
- 33 pages of references

Thanks for Your Attention!