



Advanced Topics in PID Control System Design, Automatic Tuning and Applications

Fundamentals on event-based PID control

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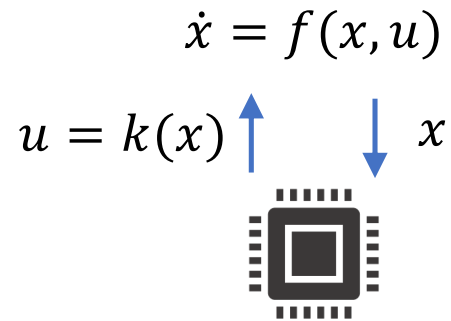
Berlin, 11 July 2020

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2. What is an event?
3. Relay control systems
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5. A two-degree-of-freedom event-based PI controller
6. Event based model identification
7. Conclusions

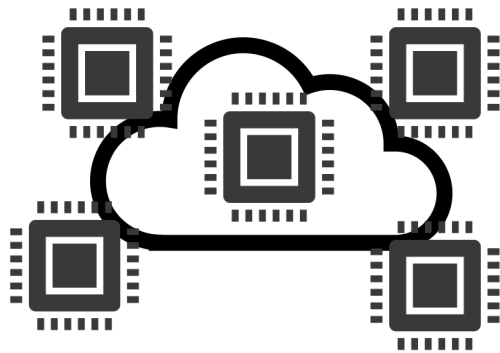
1. Introduction

Traditional control systems



...if the microprocessor has enough capacity
...if the sensors are accurate enough
...if the actuators are fast enough

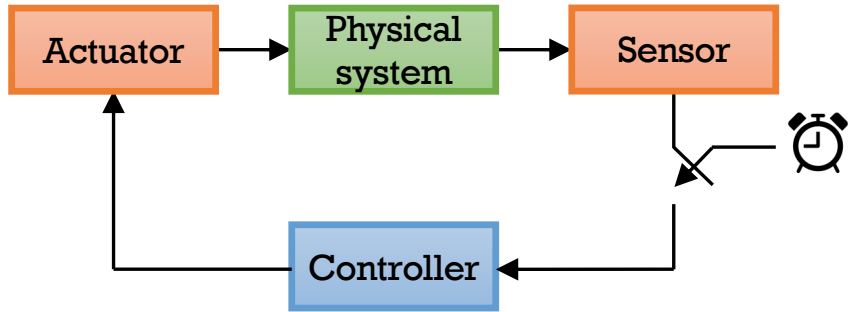
Network control systems



- Reduced communication and computing capabilities
- Limited available power

Integration of control, computing and communication

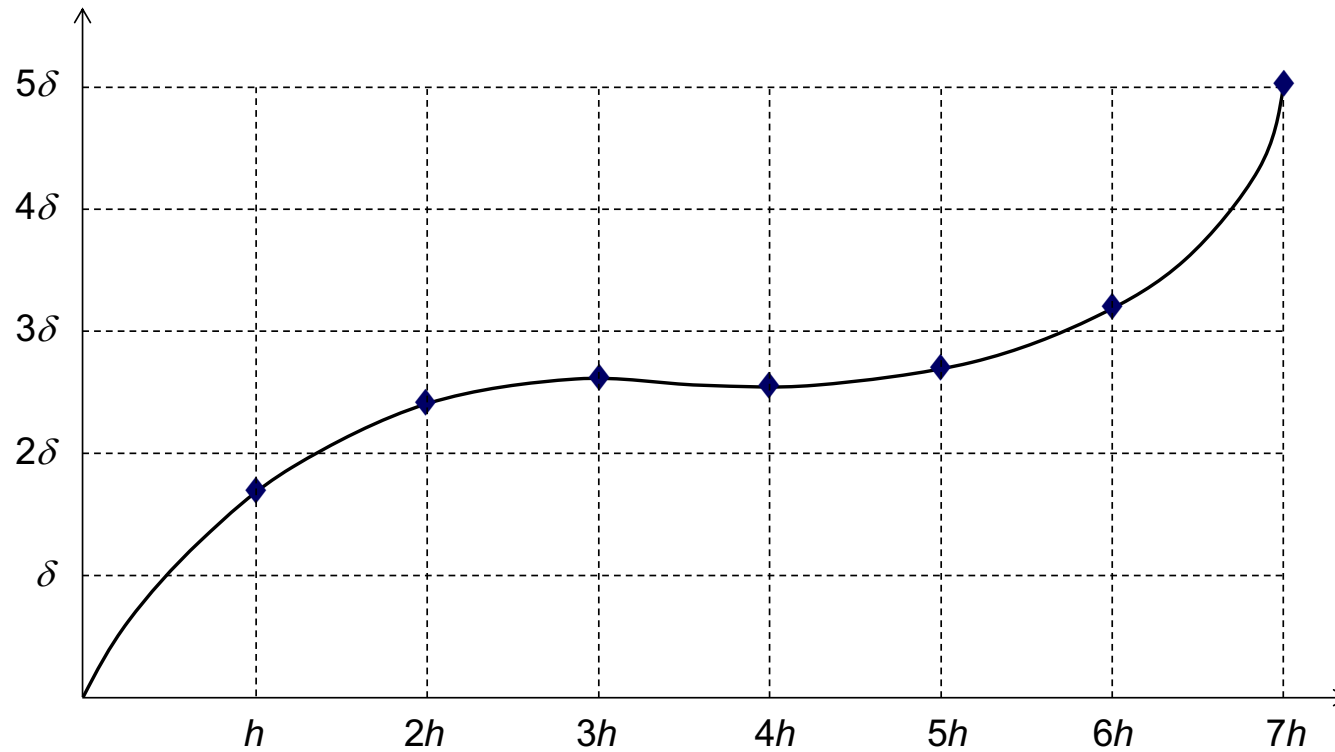
1. Introduction



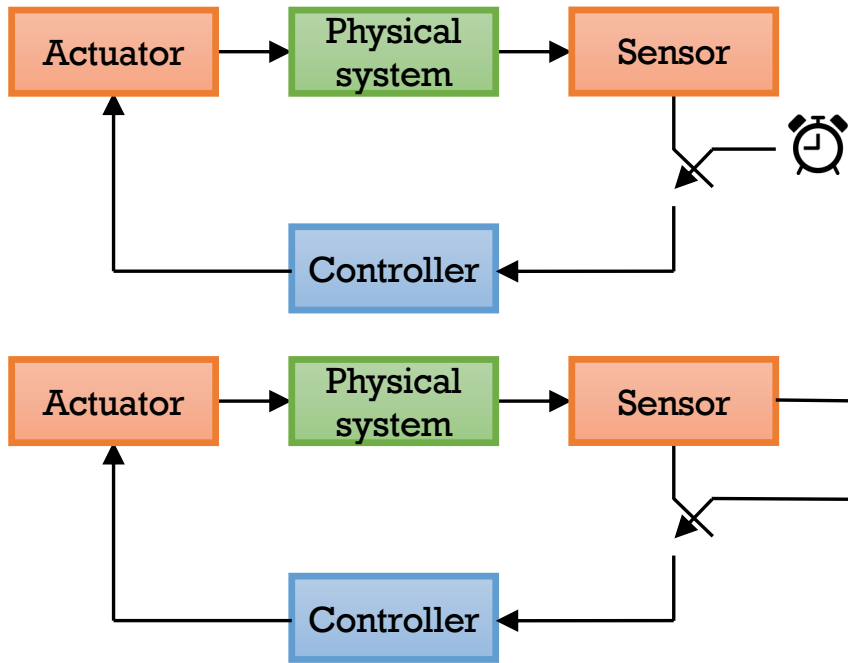
time-triggered control (TTC)

1. Introduction

Time based sampling



1. Introduction

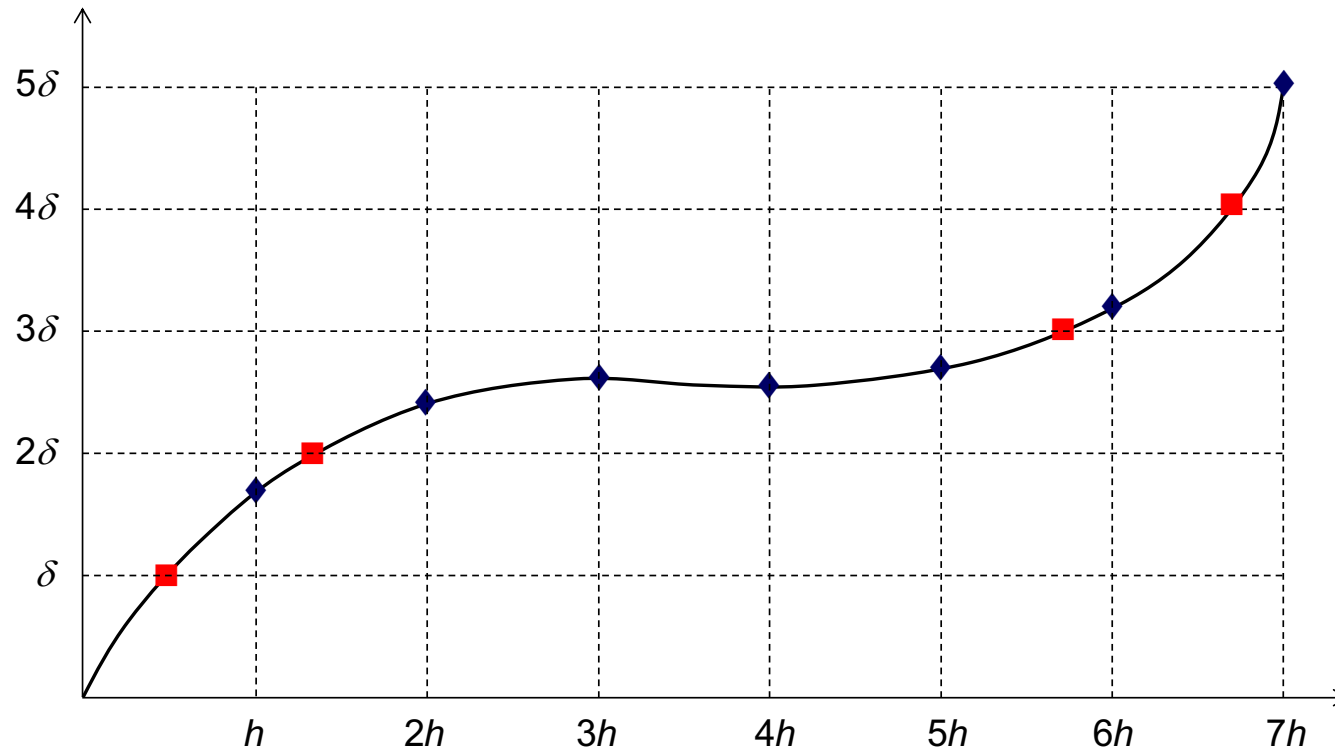


time-triggered control (TTC)

event-triggered control (ETC)

1. Introduction

Event based sampling



1. Introduction

Time based sampling versus Event based sampling

Time based sampling	Event based sampling
<ul style="list-style-type: none">▪ Represent $y(t)$ by $\{y(kh)\}$▪ Injectivity if $h < 1/f_N$ (Shannon)▪ LTI systems \Rightarrow periodic systems▪ Theory developed and mature▪ Powerful control design methods▪ Safe implementation▪ Training oriented	<ul style="list-style-type: none">▪ Represent $y(t)$ by $\{t_i\}$; $y(t_i) = n\delta$▪ Actuators event based▪ Sensors event based▪ δ-Σ, IPFM modulators▪ Process supervision (SPC)▪ Pulse feedback systems▪ Real neurons

1. Introduction

Dificulties with Time based and Event based sampling

Time based sampling	Event based sampling
<ul style="list-style-type: none">▪ Multirate sampling▪ Distributed systems: asynchronism▪ Communication networks▪ Variable delays▪ Sampling jitter▪ Biológico systems–no central clock	<ul style="list-style-type: none">▪ Very active research topic

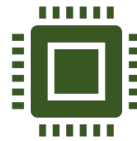
1. Introduction

Advantages of event-triggered control

Information is transmitted only when required by the system



- Minimization of energy consumed



- More processor time available



- Better adaptation to the dynamics of the process
- Better response to disturbances and overloads

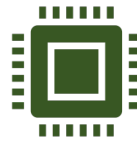
1. Introduction

Advantages of event-triggered control

Information is transmitted only when required by the system



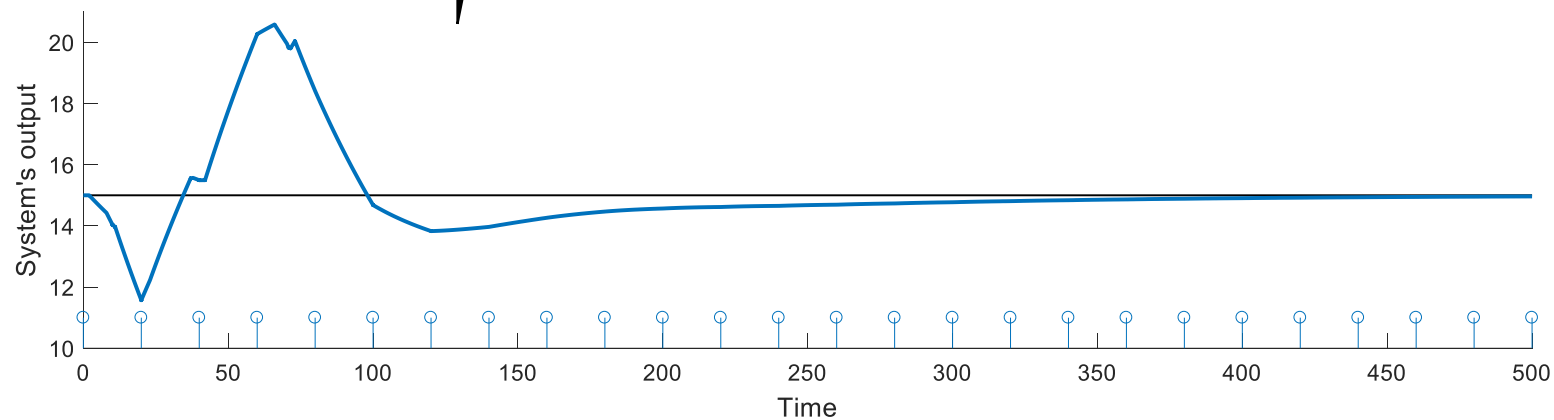
- Minimization of energy consumed



- More processor time available



- Better adaptation to the dynamics of the process
- Better response to disturbances and overloads



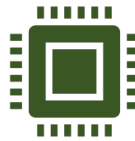
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Advantages of event-triggered control

Information is transmitted only when required by the system



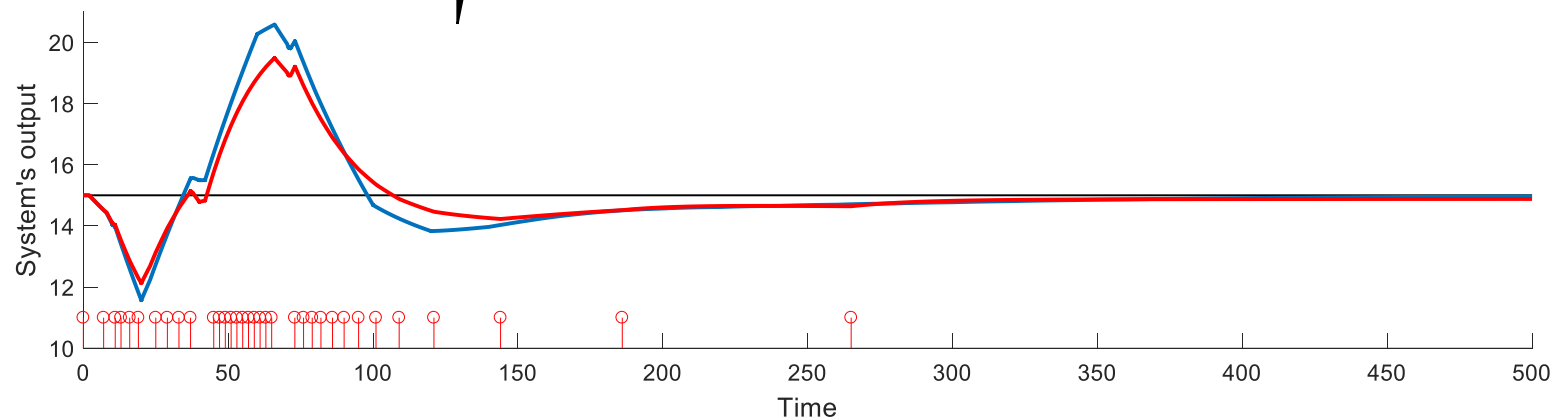
- Minimization of energy consumed



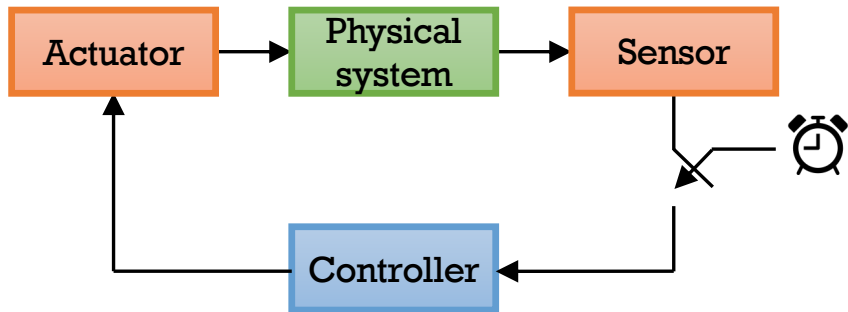
- More processor time available



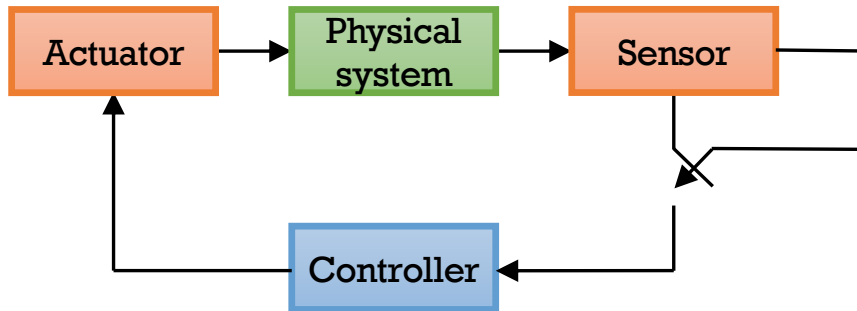
- Better adaptation to the dynamics of the process
- Better response to disturbances and overloads



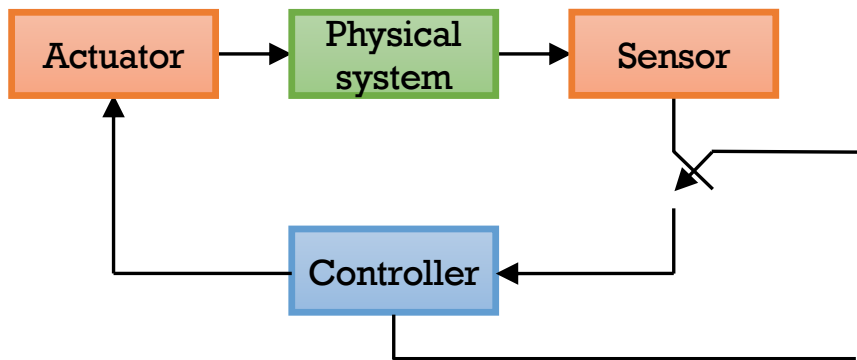
1. Introduction



time-triggered control (TTC)



event-triggered control (ETC)

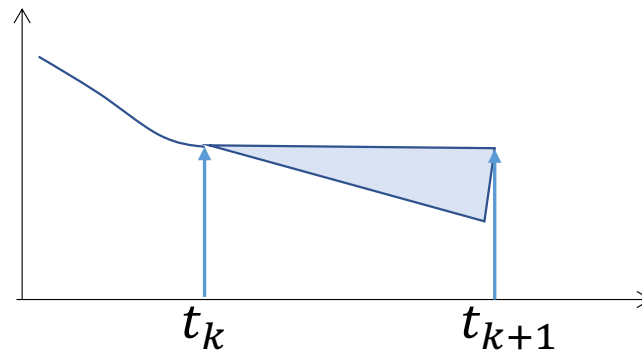


self-triggered control (STC)

1. Introduction

Self-triggered control (STC)

- **Needs:**
 - A reliable model of the system
 - More powerful computing resources
- **In the sampling instant**
 - The output of the plant is transmitted
 - The next sampling time is estimated based on the model and a robustness interval
 - Predicts future plant states
 - Predicts when the trigger condition will cross zero

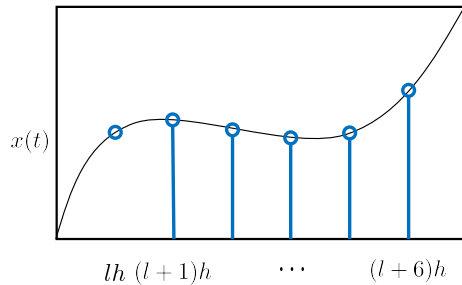


1. Introduction

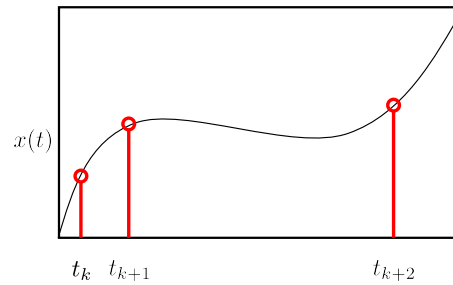
Periodic-triggered control (PTC)

The event-triggering condition is periodically evaluated

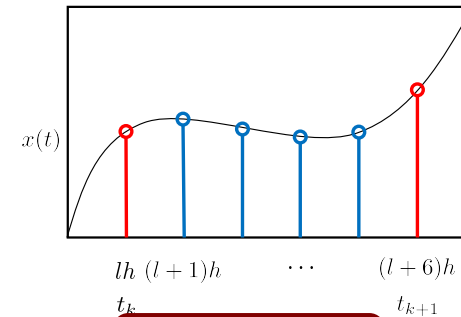
The information is aperiodically sent



TTC



ETC and STC



PETC

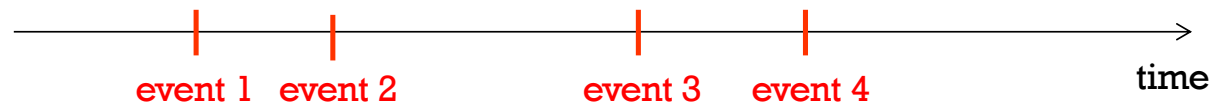
In STC t_{k+1} is calculated in t_k

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2. What is an event?

Something which occurs instantaneously at a specific time or when a specific condition occurs

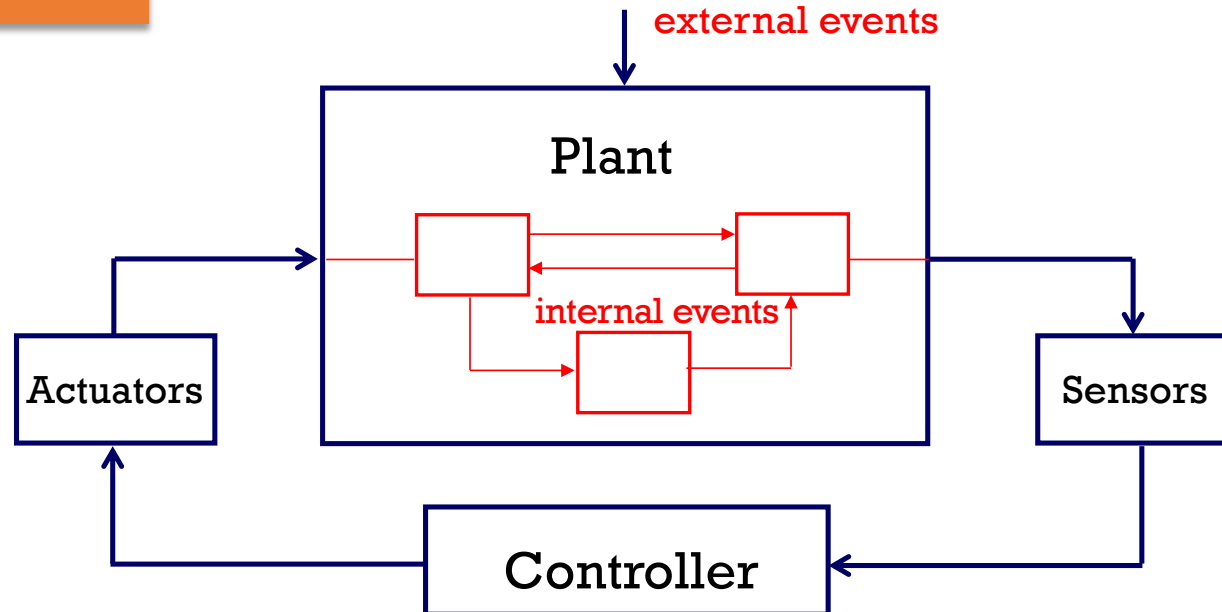


Event properties

1. Events are ordered in time (multiple events may occur concurrently)
2. Events take no time
3. There is an **event condition** for the event to happen
4. There is an **action** associated with the event

2. What is an event?

Event types



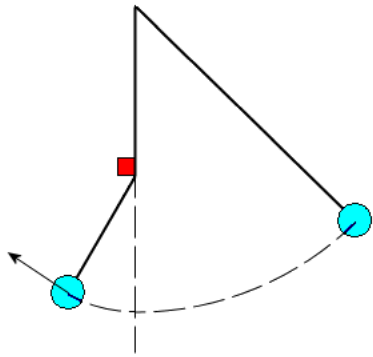
- **External:** events are related to the **input variables**
- **Internal:** events are related to internal **model variables in the plant**

time events: can be scheduled in advance (TTC)

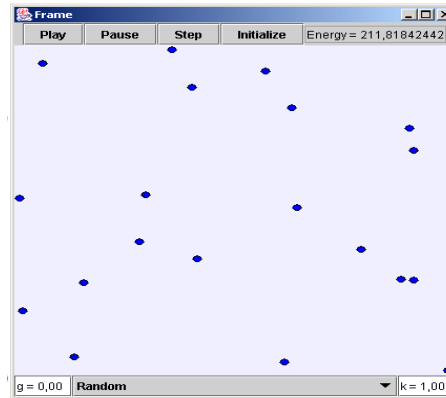
state events: cannot be scheduled in advance (ETC, STC, PETC)

2. What is an event?

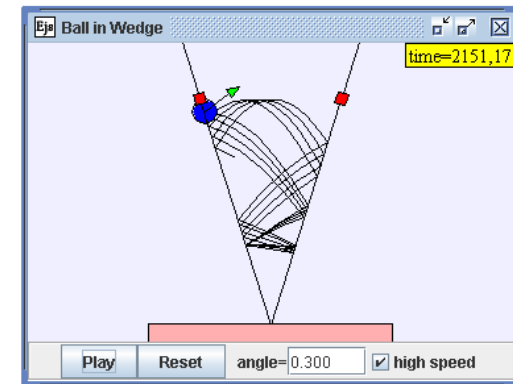
Physical examples: Events plant



Interrupted pendulum



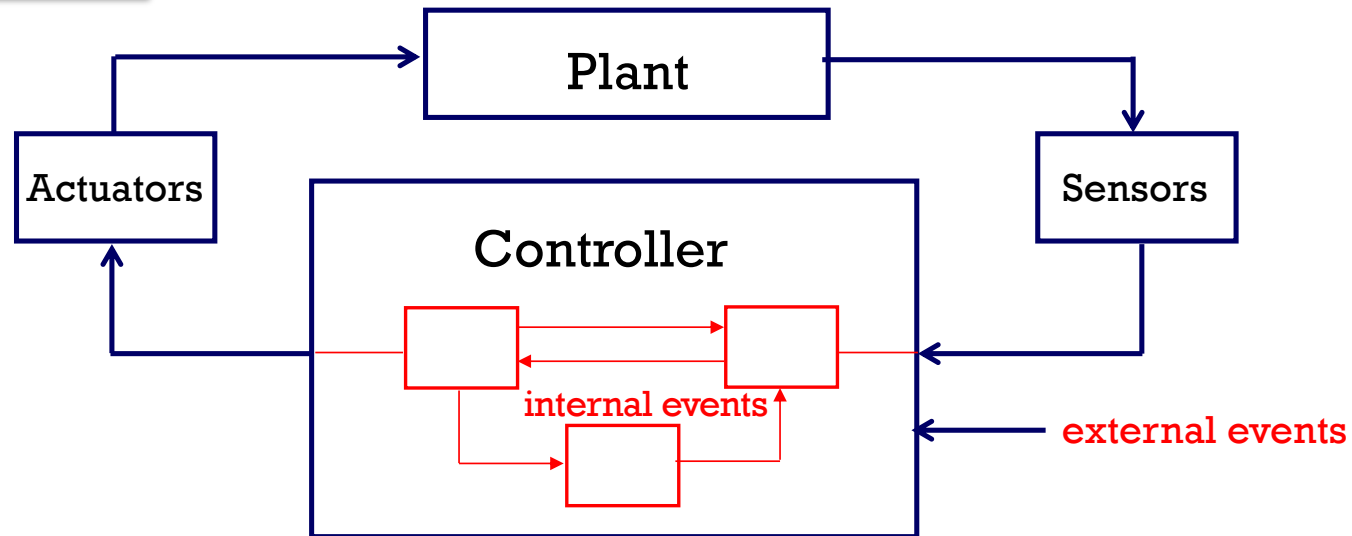
Particles collision



Ball in a wedge

2. What is an event?

Event types



- **External:** events are related to the **input variables**
- **Internal:** events are related to internal **model variables in the plant**

time events: can be scheduled in advance (TTC)

state events: cannot be scheduled in advance (ETC, STC, PETC)




2. What is an event?

How to close the loop

Monitoring the system input/output with a **trigger condition**

- Events occur when $f(\dots) \geq 0$
- After an event $f(\dots^+) < 0$
- An event is detected when f (the error) reaches a **threshold**
 - Error functions: $e(t) = x(t_k) - x(t)$, $e(t) = \hat{x}(t) - x(t)$, $V(x, t)$
 - Thresholds: c , $\|x(t)\|$, $ce^{-\alpha t}$, $S(x, t)$

When to evaluate the trigger condition

- **Continuously**  Event triggered control (ETC)
- **Periodically**  Periodically event triggered control (PETC)
- **At predetermined moments**  Self triggered control (STC)

2. What is an event?

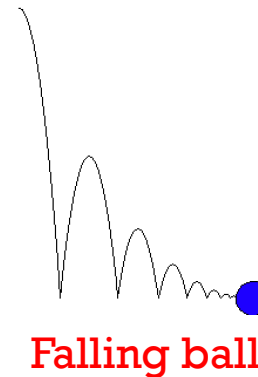
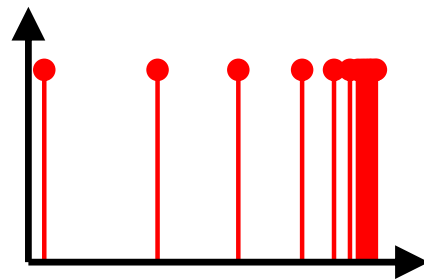
Terminology is not well established

Other terms

- Dead-zone control
- Send-on-delta control
- Intermittent control
- Controlled communication
- Adaptive sampling
- Control monitoring at any time
- Minimum attention control
- Level crossing
- ...

Zeno behavior

Occurrence of consecutive events at the same time



2. What is an event?

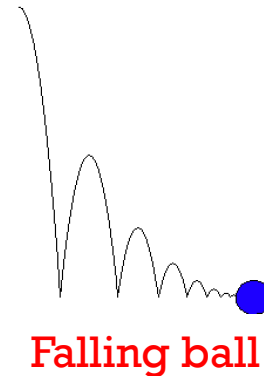
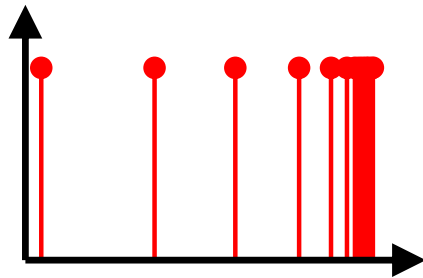
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Other terms

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- **Adaptive sampling**
- Control monitoring at any time
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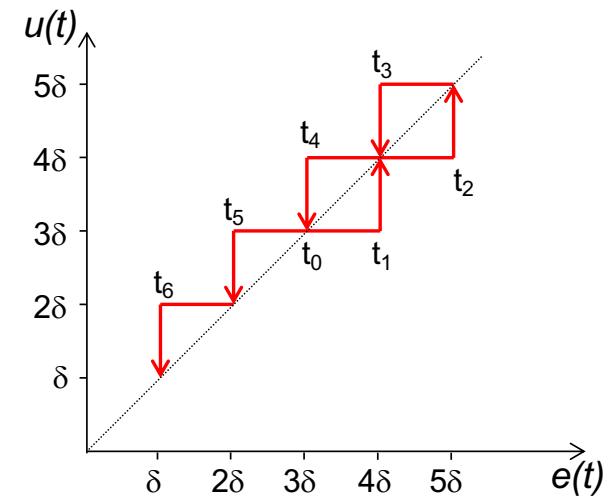
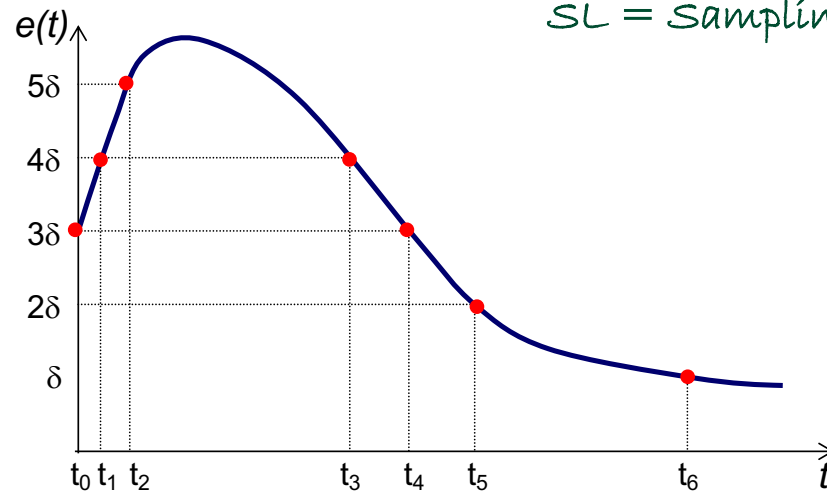
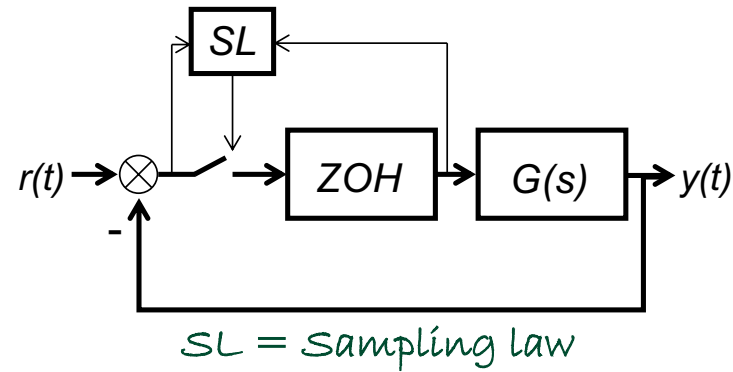
Zeno behavior

Occurrence of consecutive events at the same time



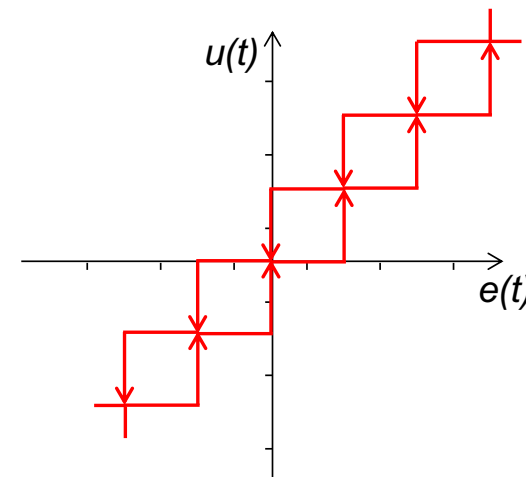
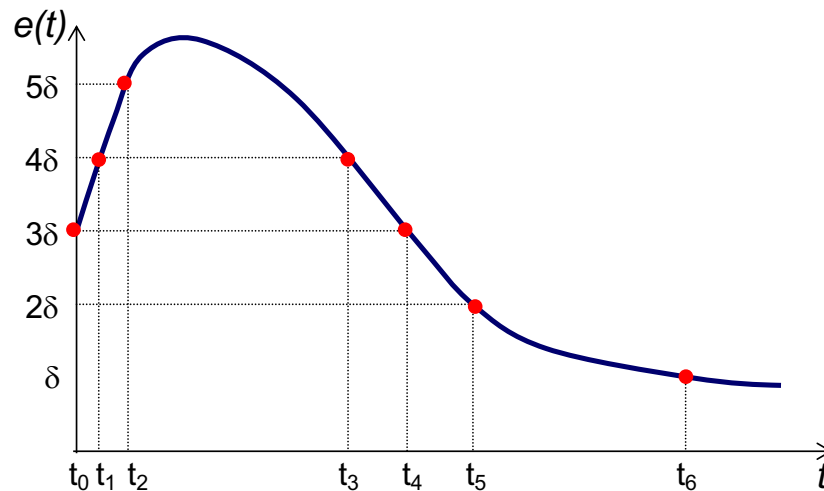
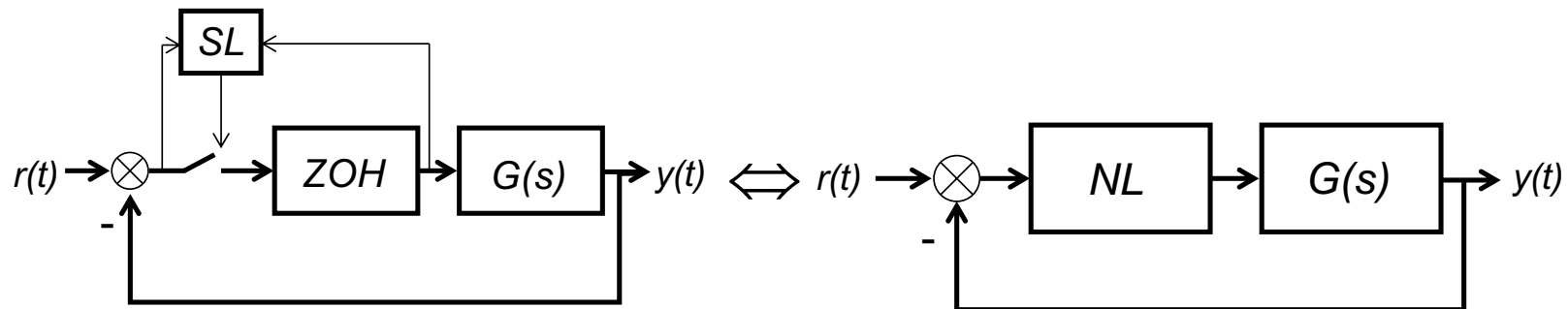
2. What is an event?

The "send-on-delta sampling" criterion: $|e(t) - e(t_i)| = \delta$



2. What is an event?

The "send-on-delta sampling" criterion: $|e(t) - e(t_i)| = \delta$



2. What is an event?

The "send-on-delta sampling" criterion: $|e(t) - e(t_i)| = \delta$

State Event Parameters

alfa: 0.4
beta: 1

Simulation Parameter

tmax: 15

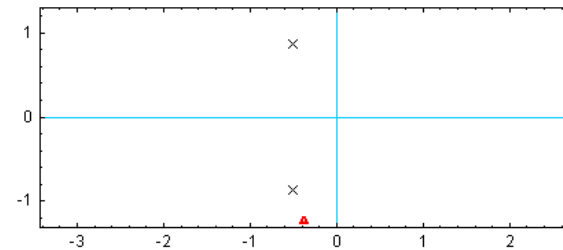
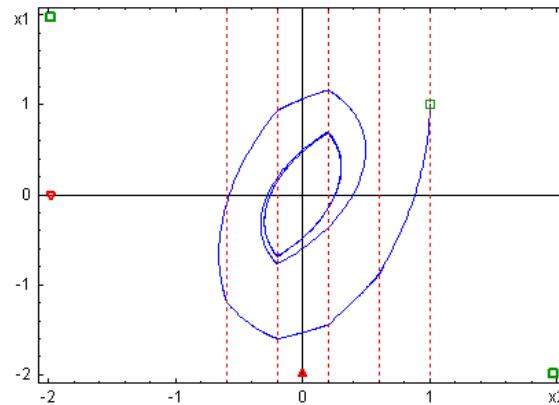
Process Parameters

Kp: 3

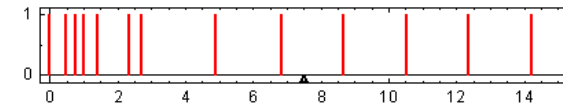
Mode: Move Add Remove
Gp(s): Poles Zeros Integrator

p: 0.500 q: 0.500

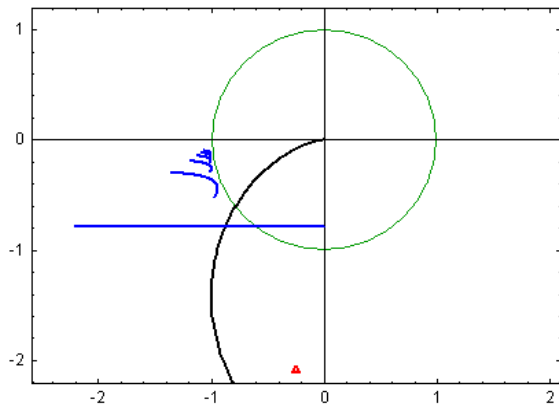
Phase Portrait



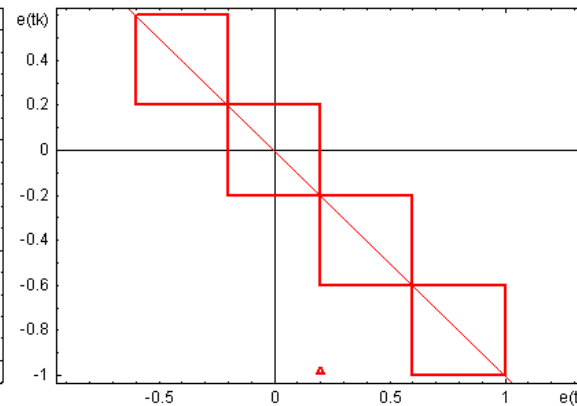
State Based Events



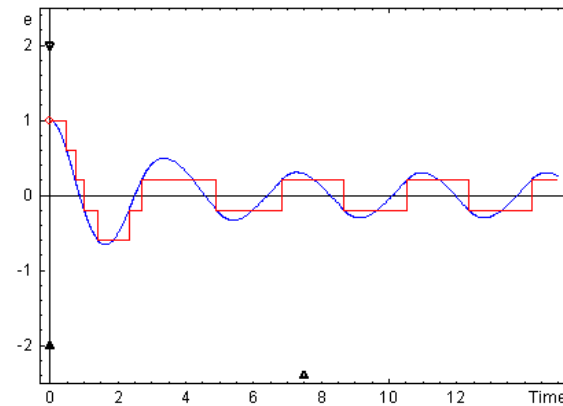
L-plane



Non Linear Element



Error



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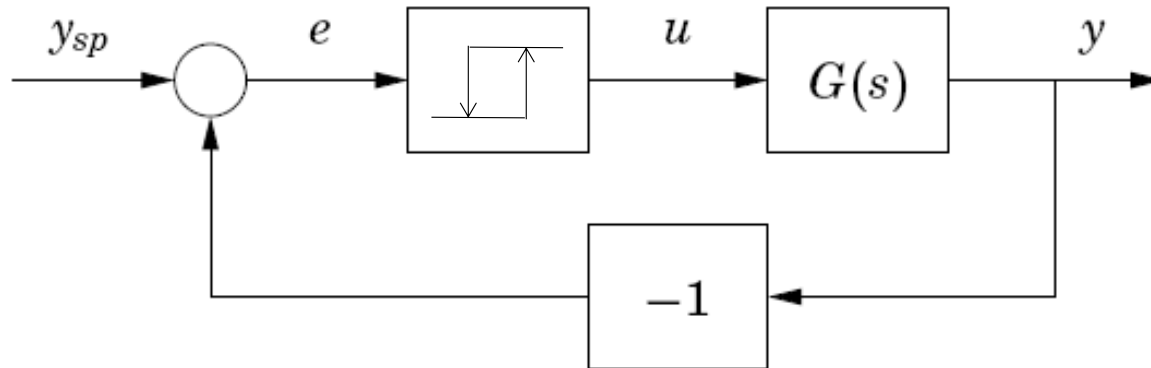
3. Relay control systems

- The **simplest** hybrid and **event based control system?**
 - Two discrete states
 - Linear continuous behavior
- Relay systems are **common**
- Relay systems has been **studied for a long time**
- Relays systems are **still widely used**
 - DC/DC converters, Relay auto-tuning, Coulomb friction
 - On-off control, D-S modulators, Variable structure systems
 - Self-oscillating adaptive systems, Amplifiers
- Relay systems **have a rich dynamic behavior**

3. Relay control systems

A challenge

To understand the behavior of the system



Some things are well known but important problems remain

Find all transfer functions such that there is a stable limit cycle

3. Relay control systems

The "relay" toolbox

Relay Parameters

hysteresis: 0.39
dead zone: 0.00
amplitude: 1.00
Td: 0.08



Simulation Parameter

tmax: 6



Process Parameters

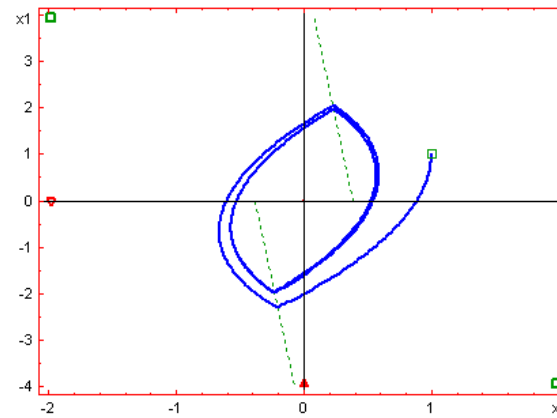
Kp: 3.00



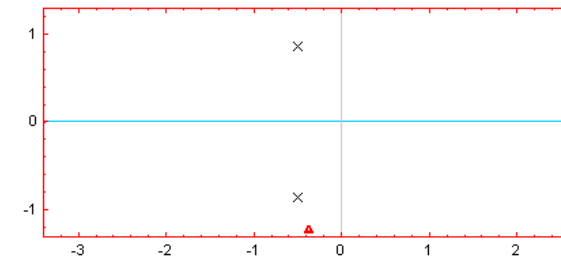
delay: 0.00

Mode: Move Add Remove
Gp(s): Poles Zeros Integrator

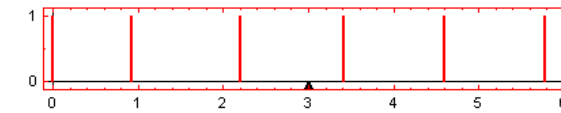
Phase Portrait



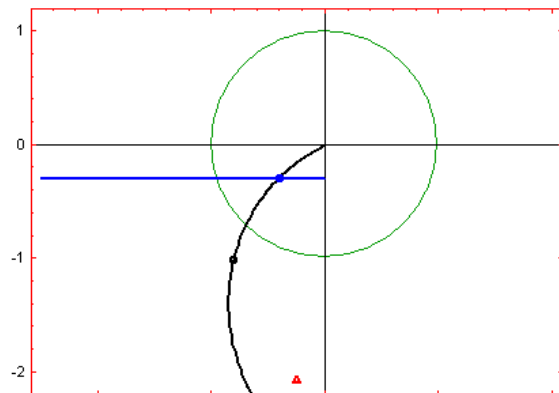
Transfer Function



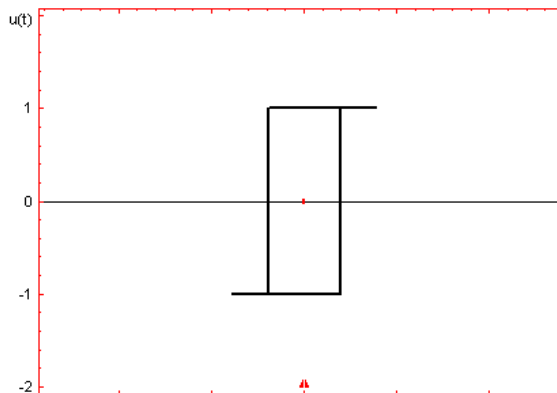
State Based Events



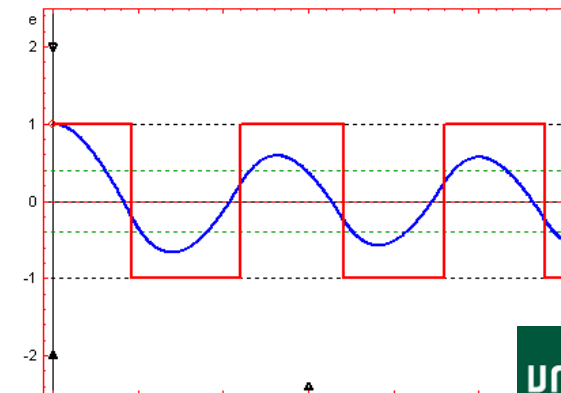
L-plane



Non Linear Element



Error + Control



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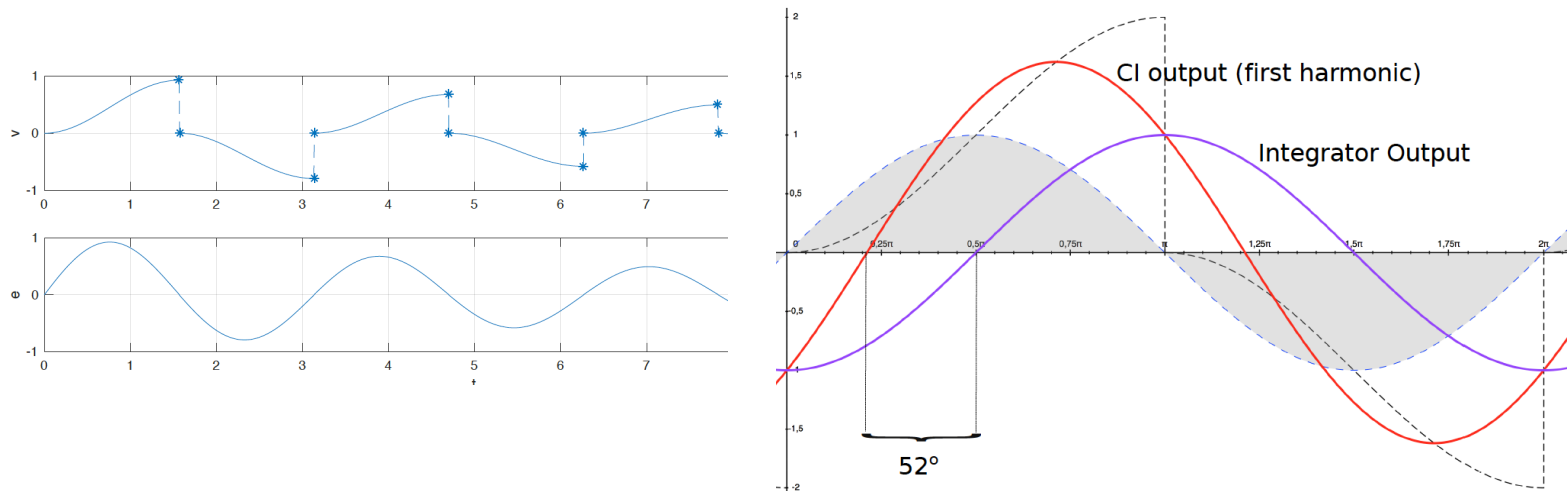
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4. Reset control systems

Clegg's integrator

A nonlinear integrator for servomechanisms

Basic idea: The integrator state/output is set to zero (reset) at those instants in which the integrator input is zero.

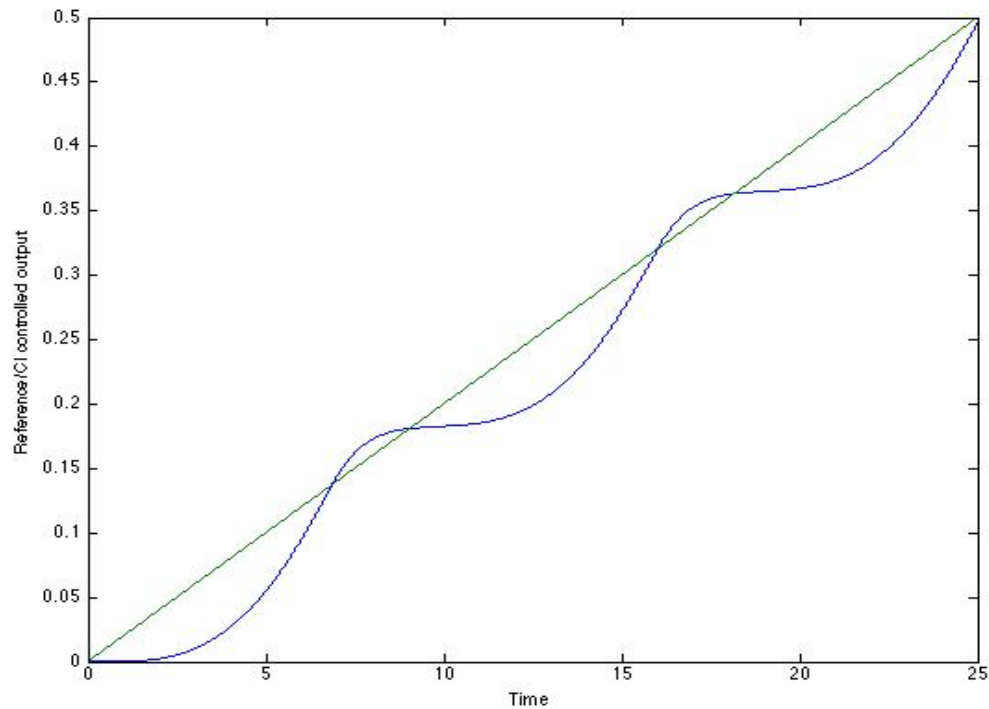


The Clegg integrator represents an attempt to synthesize a nonlinear circuit having the amplitude-frequency characteristic of a linear integrator while avoiding the 90° phase lag associated with the linear transfer function.

4. Reset control systems

Clegg's integrator

...but asymptotic properties of the integrator are lost!

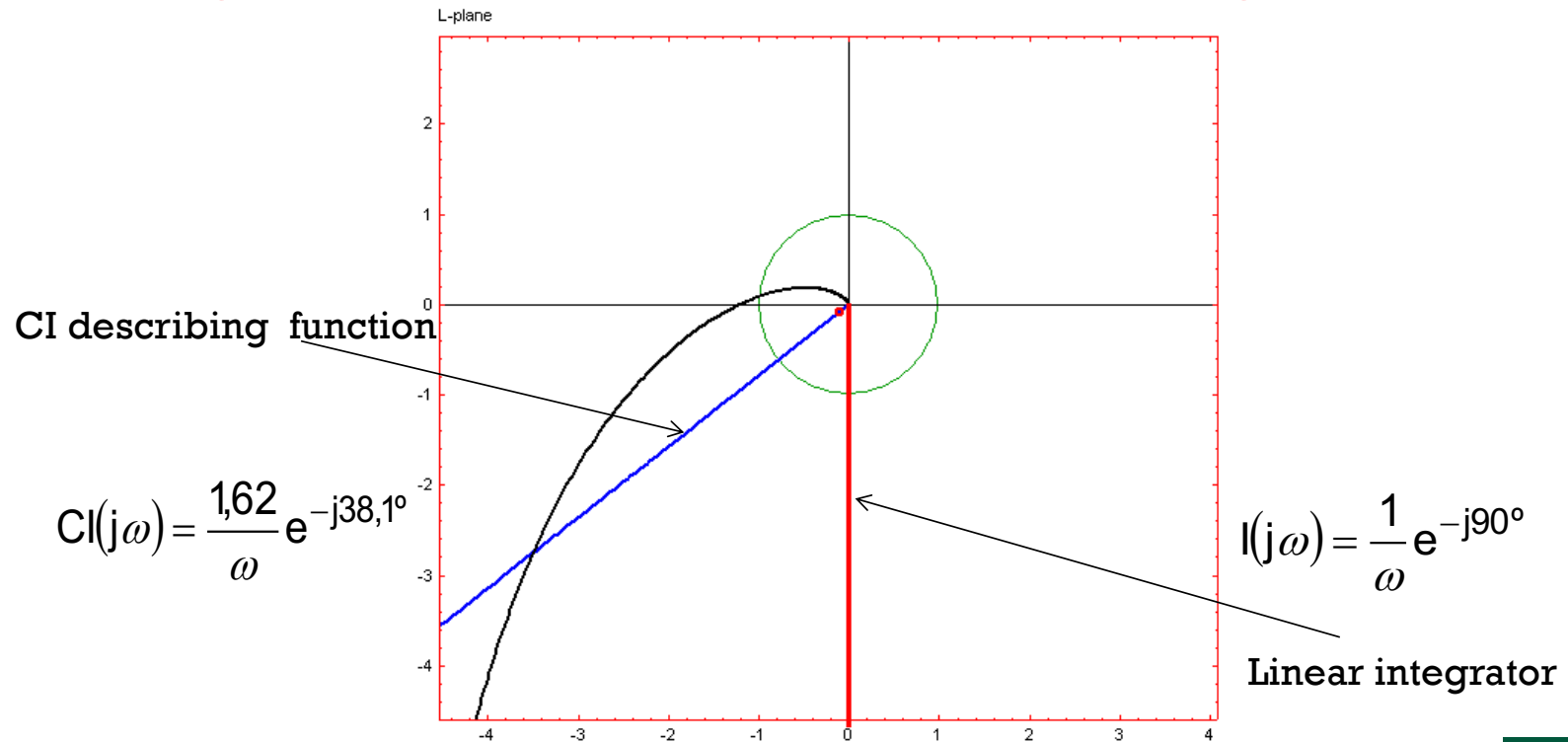


J.C. Clegg, A nonlinear integrator for servomechanisms, *Trans. A. I. E. E. m*, Part II 77 (1958) 41–42

4. Reset control systems

Clegg's integrator

CI gives phase lead over a (linear) integrator



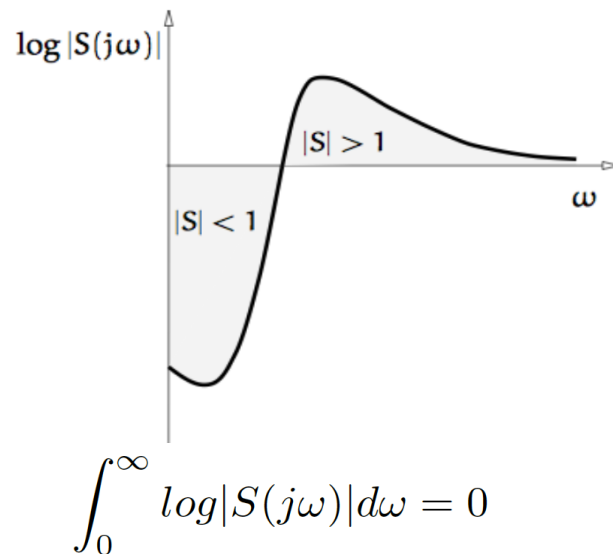
4. Reset control systems

Main motivation of reset control systems

Overcoming fundamental limitations of LTI controllers

Frequency domain:

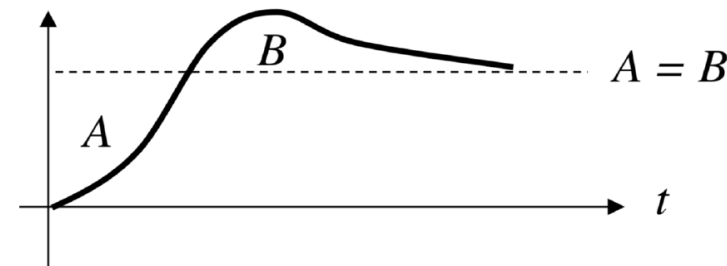
The “area formula” (Bode)



(for $L(s)$ with poles-zeros excess of 2 or more, and no open-loop poles in RHP)

Time domain:

Another “area formula”



$$\int_0^{\infty} e(t) dt = 0$$
$$\lim_{t \rightarrow \infty} e(t) = 0$$

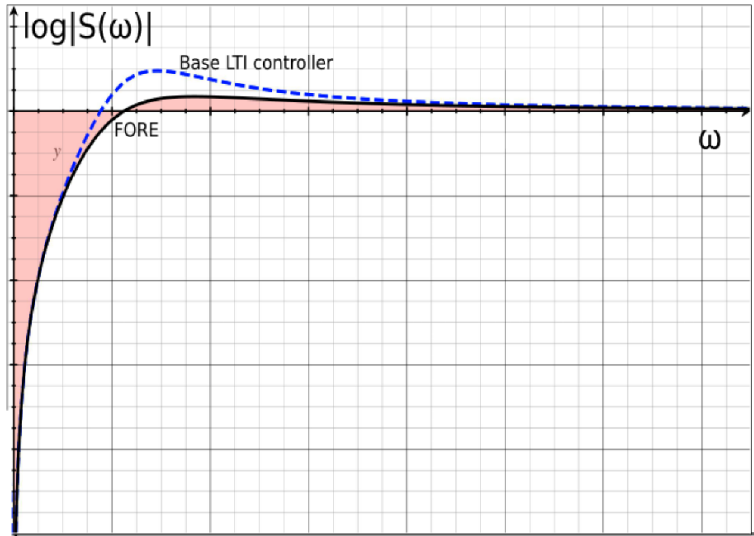
(for $L(s)$ having 2 or more integrators)

4. Reset control systems

Main motivation of reset control systems

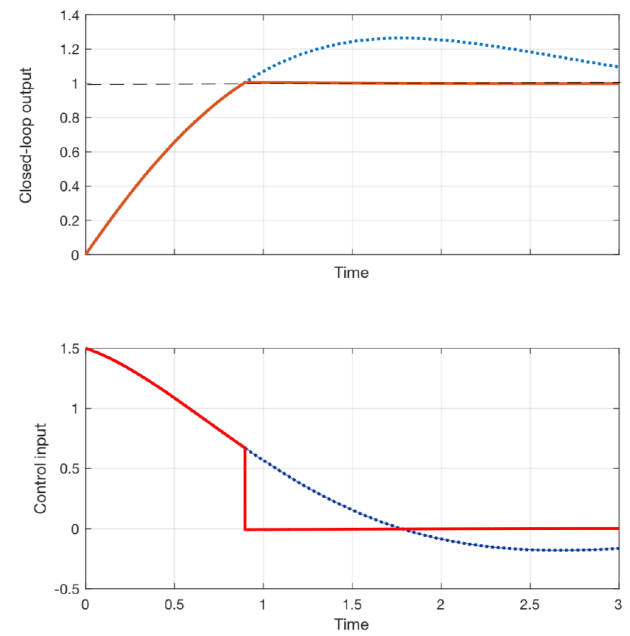
Overcoming fundamental limitations of LTI controllers

Frequency domain



$$\int_0^\infty \log|S(j\omega)|d\omega < 0 \text{ (FORE)}$$
$$\int_0^\infty \log|S(j\omega)|d\omega \geq 0 \text{ (any LTI controller)}$$

Time domain

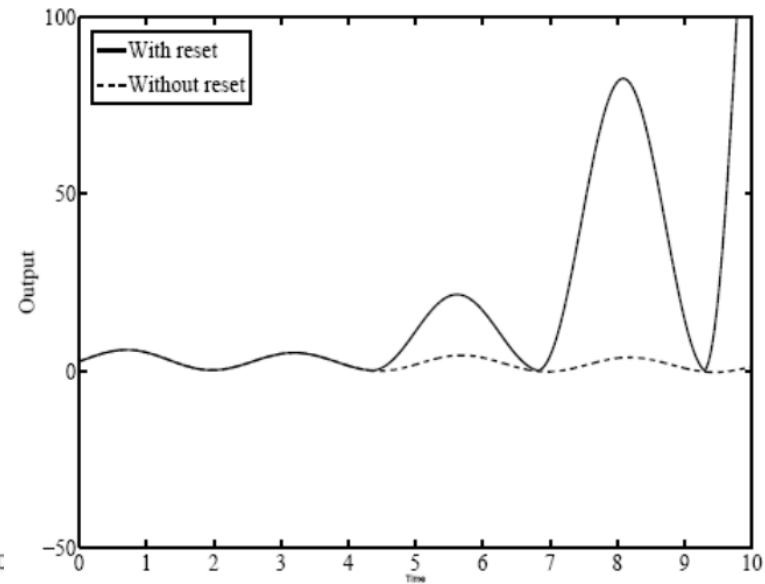
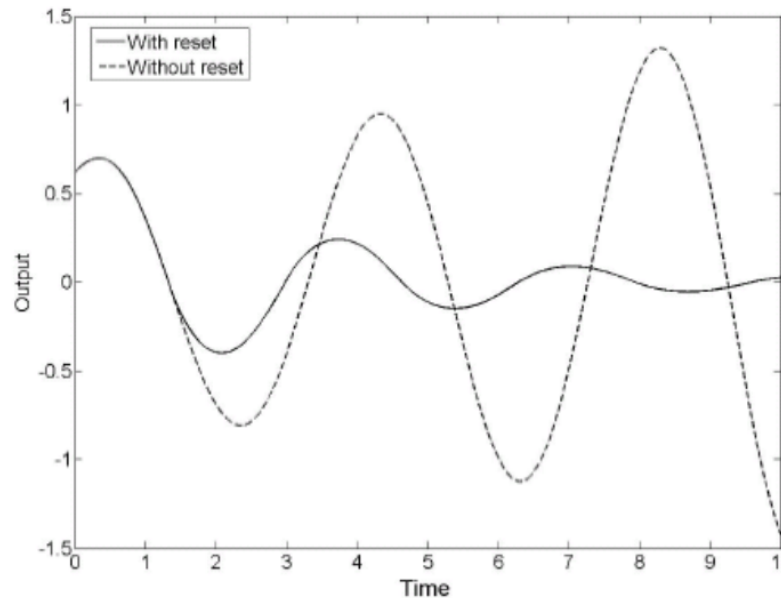


No overshoot (even for fast responses) !
LTI controllers (loop gain with 2 integrators) always produce overshoot !

4. Reset control systems

Stability

The problem is non-trivial: reset can stabilize an unstable base system,
... But also can destabilize a stable base system!

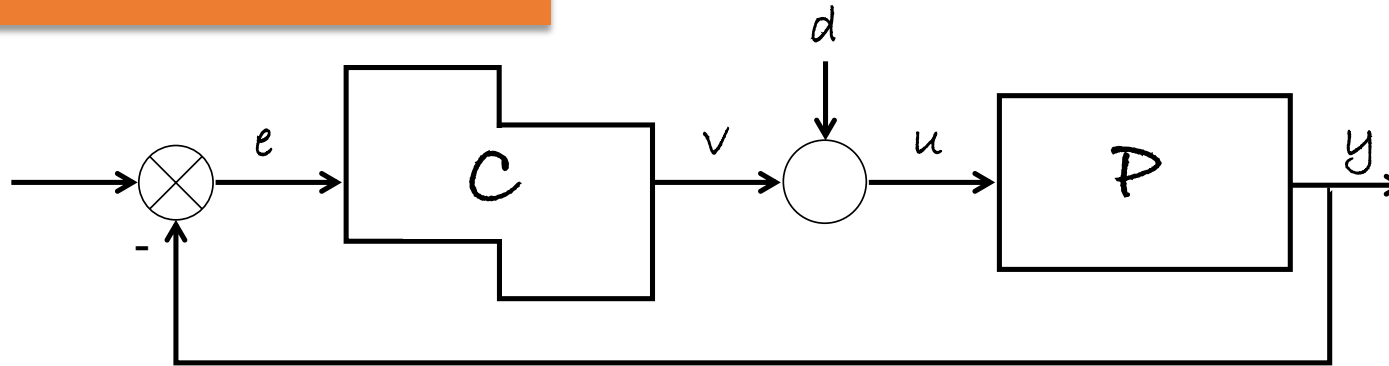


Some sufficient conditions for stability:

- independent on the reset instants: H_β -condition (Bekker-Hollot-Chait'2000)

4. Reset control systems

Stability: Problem formulation



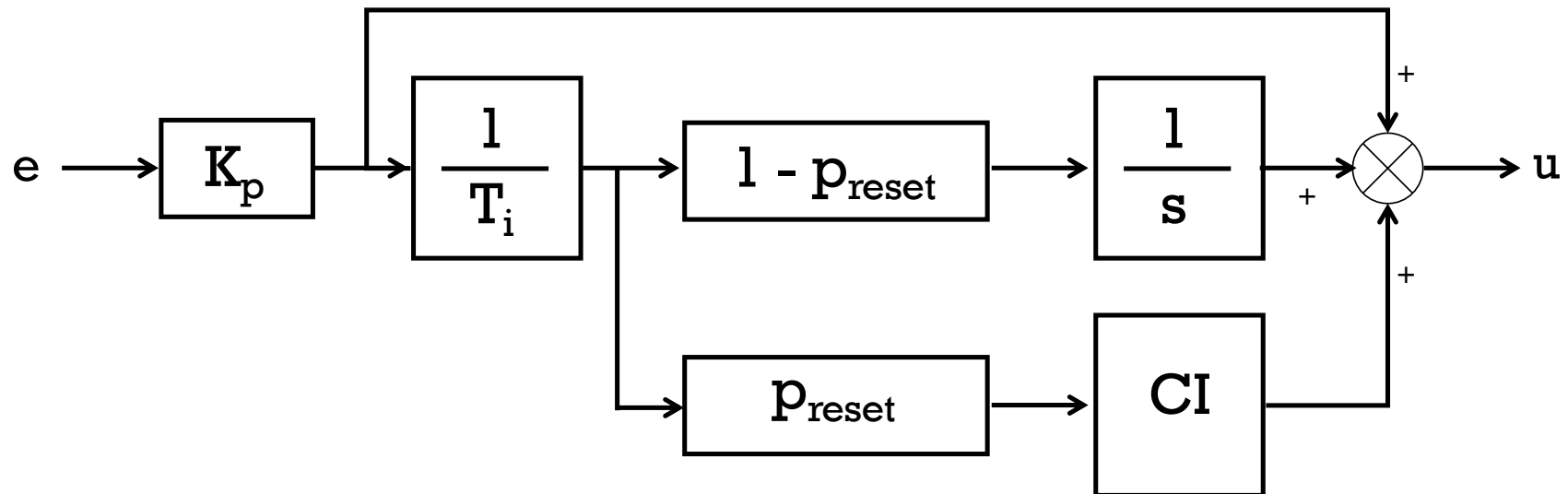
$$C: \begin{cases} \dot{x}_r = A_r x_r + B_r e, & e \neq 0 \\ x_r^+ = A_\rho x_r, & e = 0 \\ v = C_r x_r \end{cases}$$

$$P: \begin{cases} \dot{x}_p = A_p x_p + B_p u \\ y = C_p x_p \\ u = v + d \end{cases}$$

- The base system is LTI
- The reset instants are defined as the time crossings of the error signal with zero
- Zeno solutions may exist, but are easily removed by time regularization
- Reset actions are events dependent on the (plant and compensator) state

4. Reset control systems

PI + CI controller



4. Reset control systems

PI + CI controller: Main characteristics

- **A simple structure** easily implementable, with few parameters
- Application target: **process control**
- Good transitory and steady state properties
- LTI base compensator : **PI Antiwindup behavior**
- “Simple” **tuning rules**
 - Tune the base PI controller
 - Select the reset percentage to reduce overshoot
- A fast response with no excessive overshoot may be obtained, overcoming LTI compensation limitations.
- **Very intuitive** for manual tuning: reset is a single parameter p_{reset}
- CI: **A “derivative” action** without increasing the cost of feedback

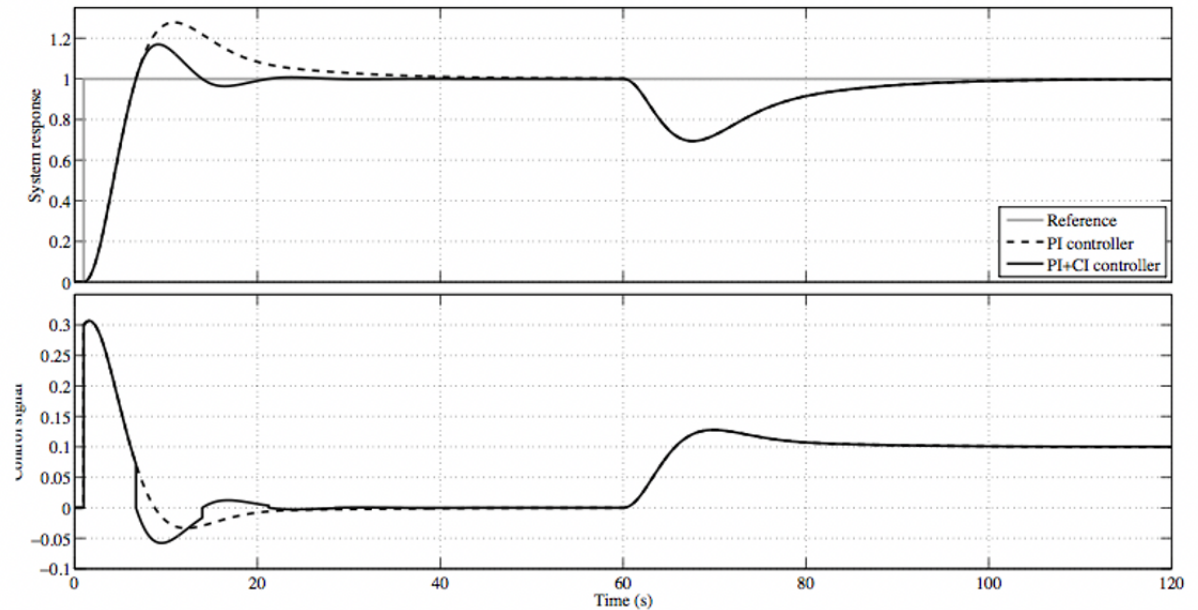
4. Reset control systems

PI + CI controller: Example IPTD system

$$\tilde{P}(s) = \frac{1}{s} e^{-1.69s}$$

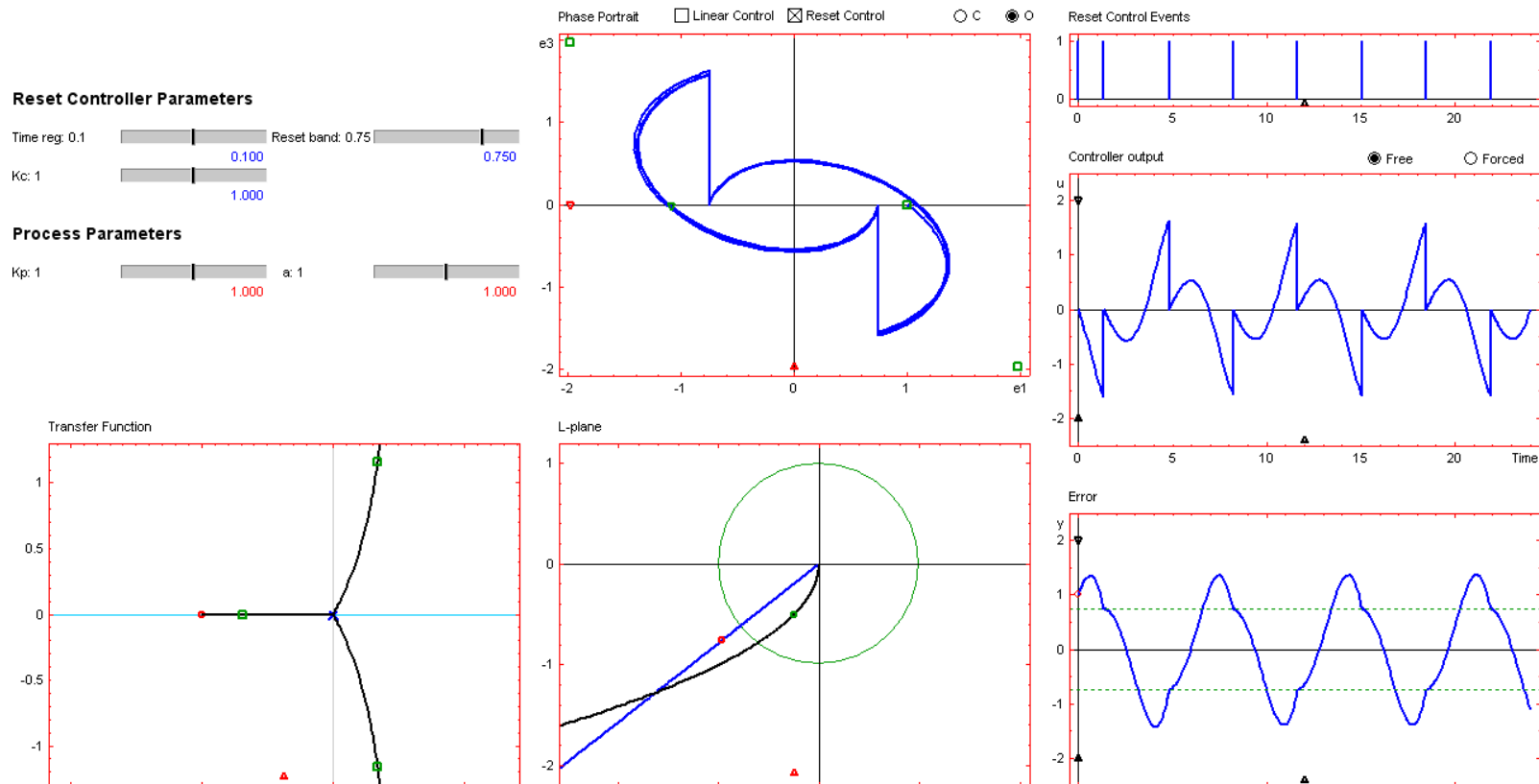
PI base (SIMC): $k_p = 0.3, T_i = 13.5 \text{ s}$

	Reference		Disturbance		Stability margins	
	IAE (s)	ITAE (s ²)	IAE (s)	ITAE (s ²)	ϕ_m (°)	A_m (dB)
PI	6.43	62.80	4.48	328.3	48.6	24.28
PI+CI	4.17	20.44	4.48	328.3	48.4	23.9



4. Reset control systems

The “reset control” toolbox

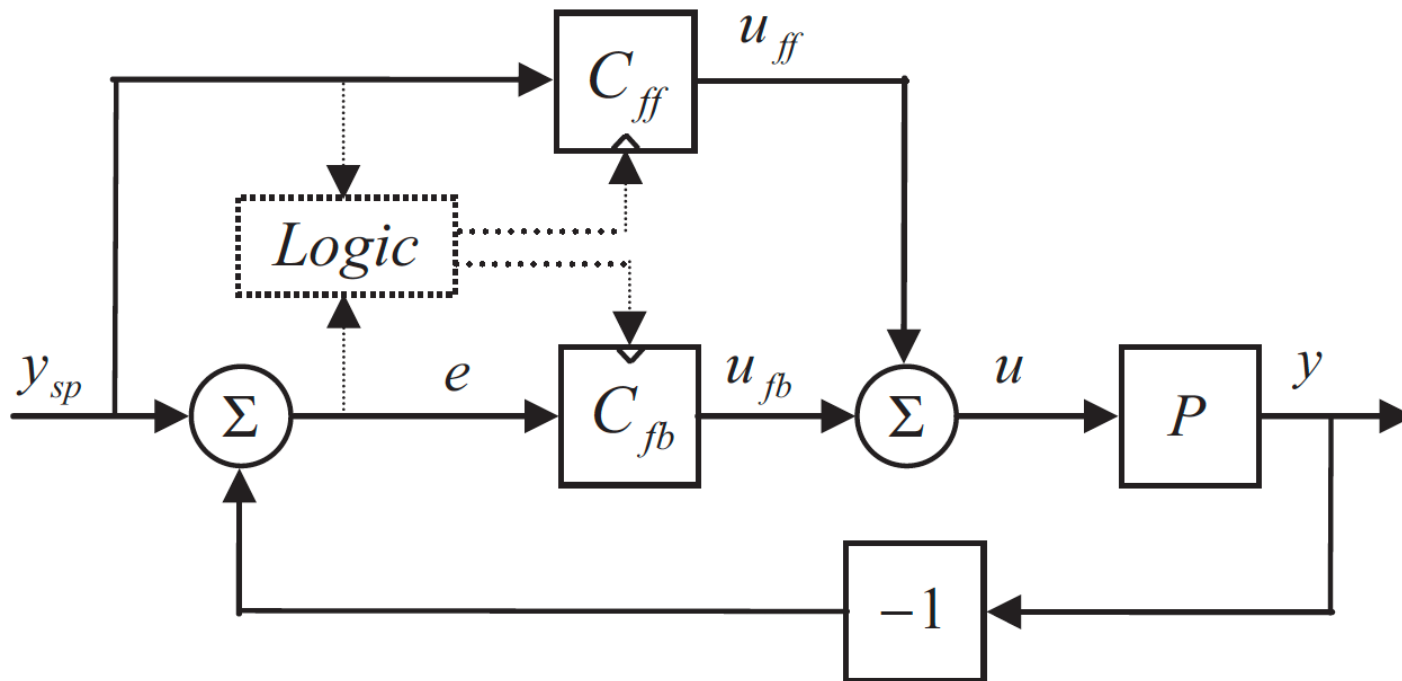


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5. **A two-degree-of-freedom event-based PI controller**
6. Event based model identification
7. Conclusions

5. A two-degree-of-freedom event-based PI controller

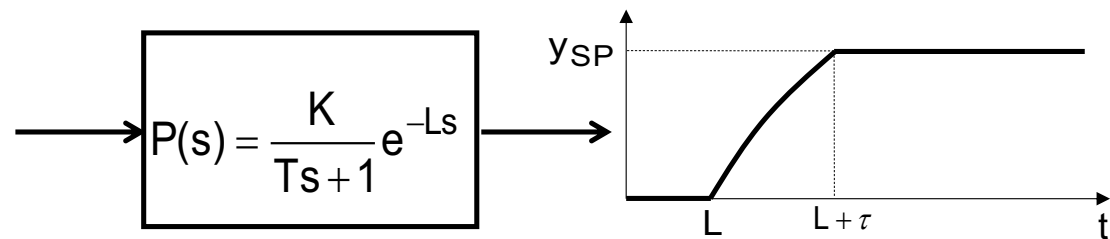
Block diagram of the event-based controller with two degrees of freedom



5. A two-degree-of-freedom event-based PI controller

Design of the event-based feed-forward compensator C_{ff}

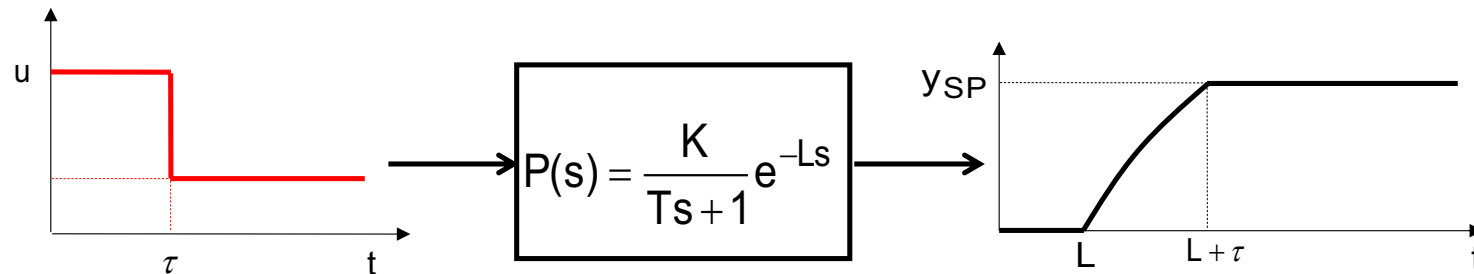
The process to be controlled is modeled as a FOPTD



5. A two-degree-of-freedom event-based PI controller

Design of the event-based feed-forward compensator C_{ff}

The process to be controlled is modeled as a FOPTD



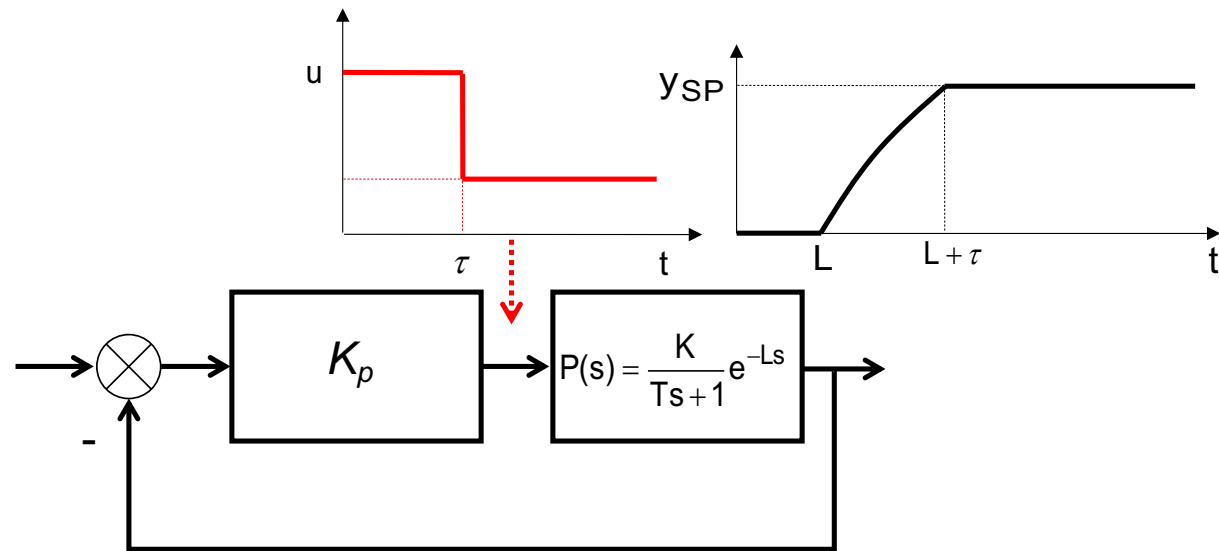
In this case, the following open-loop control action can be applied

$$u_{ff}(t) = \begin{cases} \bar{u}_{ff} & \text{if } t < \tau \\ \frac{y_{sp}}{K} & \text{if } t \geq \tau \end{cases} \quad \bar{u}_{ff} = \frac{y_{sp}/K}{1 - e^{-\tau/T}}$$

5. A two-degree-of-freedom event-based PI controller

Proportional controller

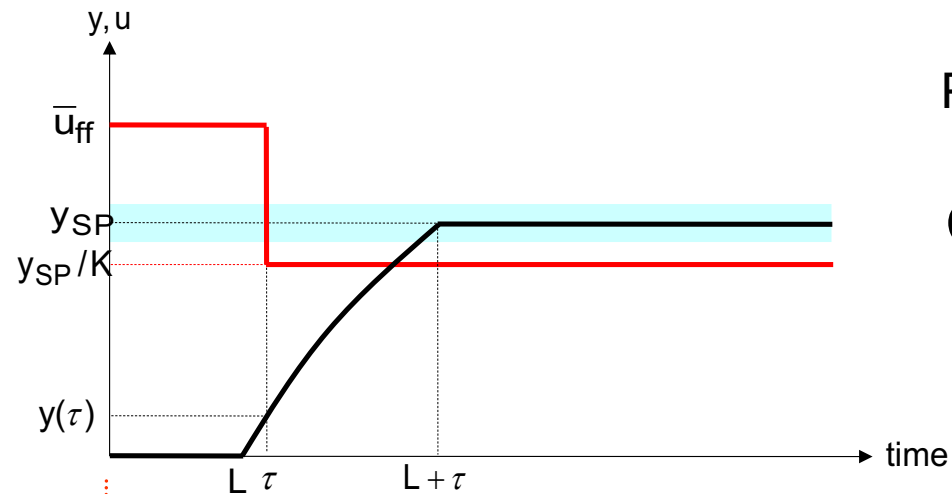
The process to be controlled is modeled as a FOPTD



$$u_{ff}(t) = \begin{cases} \bar{u}_{ff} & \text{if } t < \tau \\ \frac{y_{sp}}{K} & \text{if } t \geq \tau \end{cases} \quad \bar{u}_{ff} = \frac{y_{sp}/K}{1 - e^{-\tau/T}} \Rightarrow \tau = -T \log \left(1 - \frac{y_{sp}}{K \bar{u}_{ff}} \right)$$

5. A two-degree-of-freedom event-based PI controller

The process to be controlled is modeled as a FOPTD



$$P(s) = \frac{K}{Ts + 1} e^{-Ls}$$

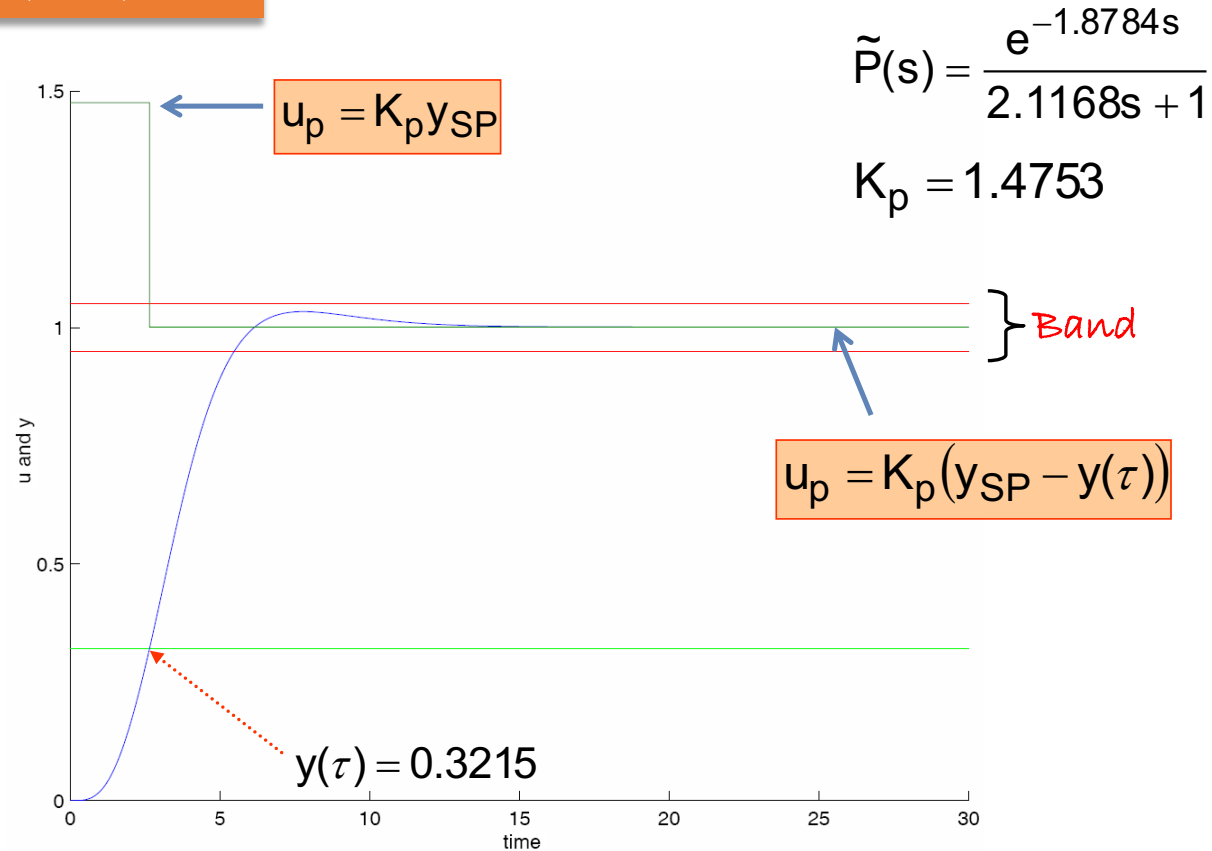
$$C(s) = K_p$$

$$u_p = K_p y_{SP}$$

$$u_p = K_p (y_{SP} - y(\tau))$$

5. A two-degree-of-freedom event-based PI controller

Example: $P(s) = (s+1)^{-4}$



5. A two-degree-of-freedom event-based PI controller

Algorithm of the integral action

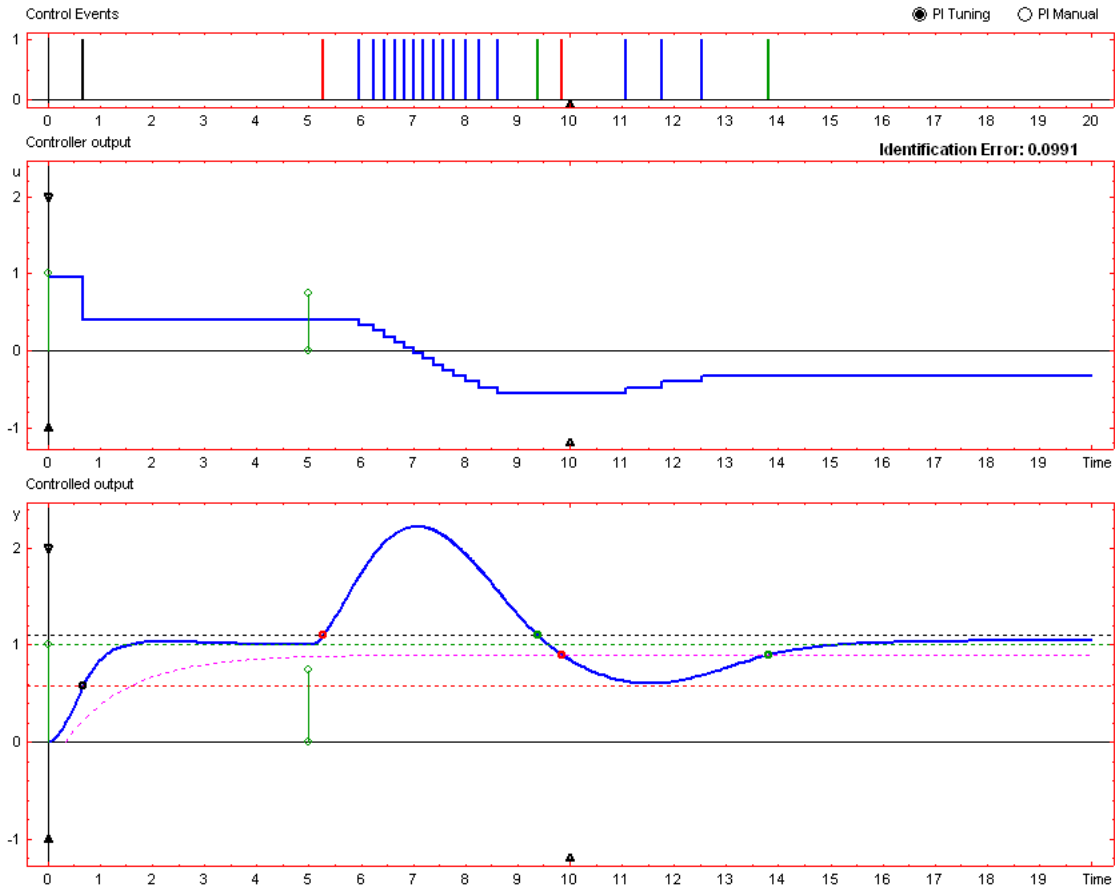
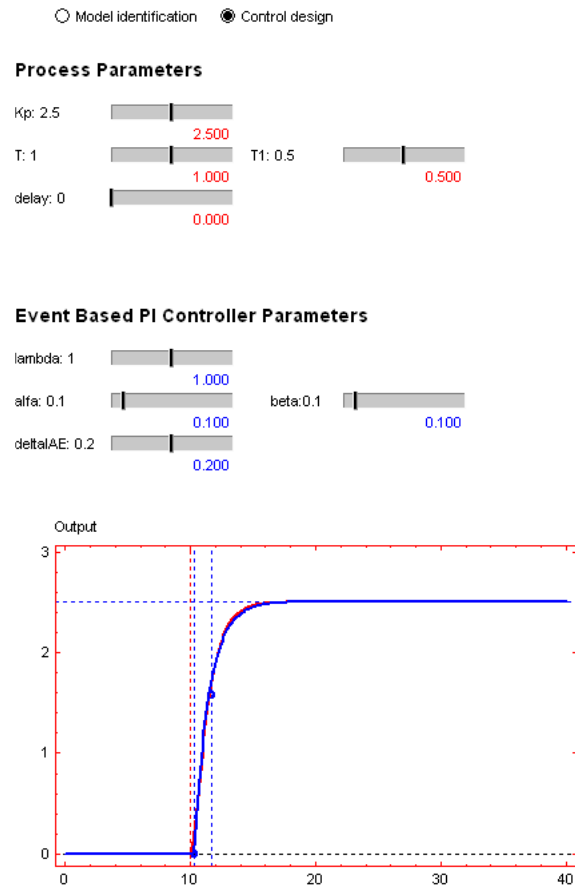
- Once the process is inside the dead-band, defined as:

$$ay_{SP} \leq y_{SP} \leq by_{SP}$$

- The proportional action is constant: $u_p = K_p(y_{SP} - D_1)$
- The integrator is enabled.
- When the process leaves the dead-band:
 - The integrator starts calculating the IAE
 - Every time the IAE exceeds a certain threshold, that is, $IAE \geq \delta_{IAE}$, it produces an integral control action.
- When the process is inside the dead-band:
 - The integrator stops working

5. A two-degree-of-freedom event-based PI controller

The “event based PI” toolbox

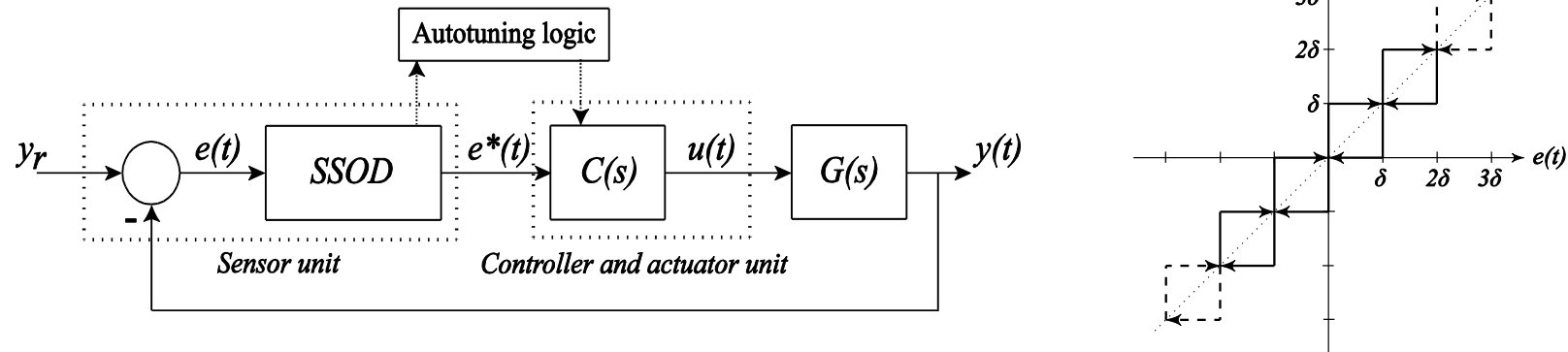


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6. Event based model identification

Identification based on the DF of the SSOD



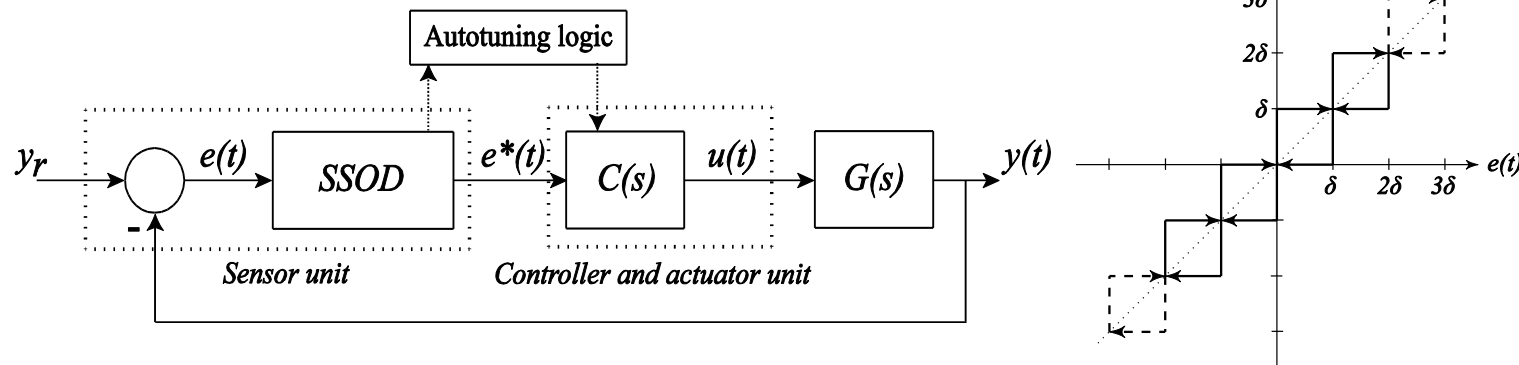
Architecture of the event-based control loop Relationship between $e(t)$ and $e^*(t)$.

The behaviour of the symmetric send-on-delta sampler can be described as:

$$e^*(t) = \begin{cases} (i+1)\delta & \text{if } e(t) \geq (i+1)\delta \text{ and } e^*(t^-) = i\delta \\ i\delta & \text{if } e(t) \in [(i-1)\delta, (i+1)\delta] \text{ and } e^*(t^-) = i\delta \\ (i-1)\delta & \text{if } e(t) \leq (i-1)\delta \text{ and } e^*(t^-) = i\delta \end{cases}$$

6. Event based model identification

Identification based on the DF of the SSOD



Architecture of the event-based control loop

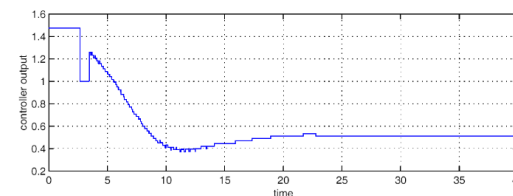
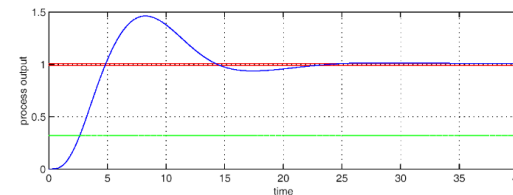
Relationship between $e(t)$ and $e^*(t)$.

In an event control system, events occur due to:

1. Changes in the set point.
2. Disturbances
3. Periodically for safety.

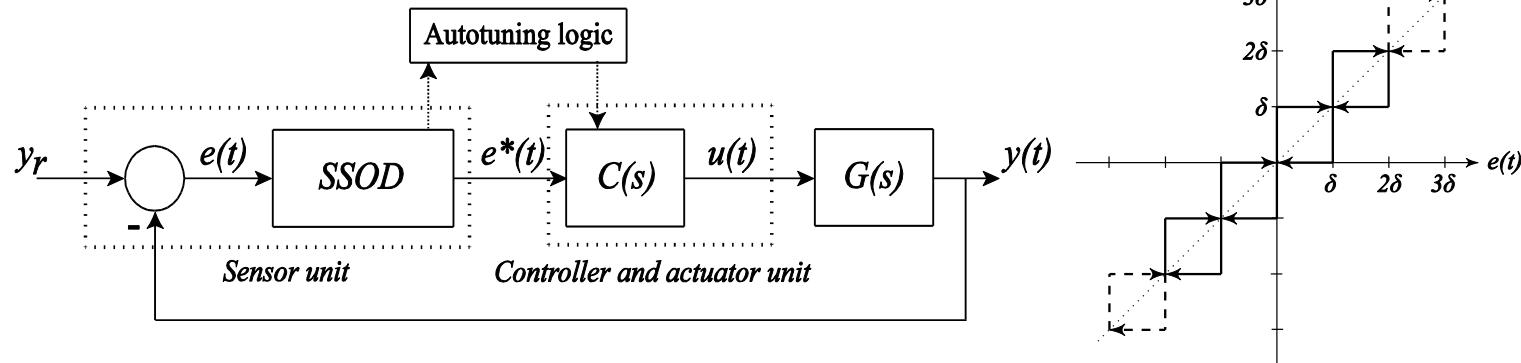
Rest of the time:

- There are no control actions.
- It should not oscillate if it is "fine" tuned.



6. Event based model identification

Identification based on the DF of the SSOD



Architecture of the event-based control loop

Relationship between $e(t)$ and $e^*(t)$.

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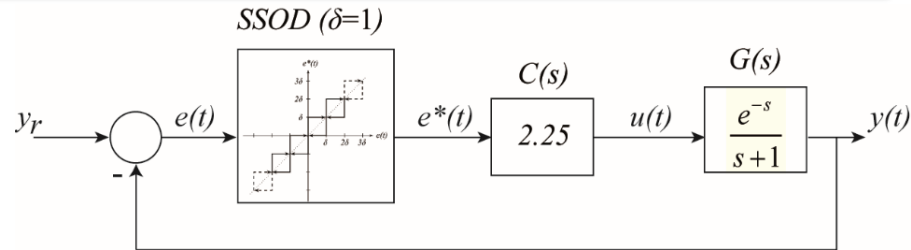
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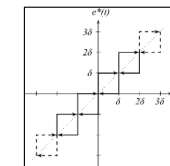
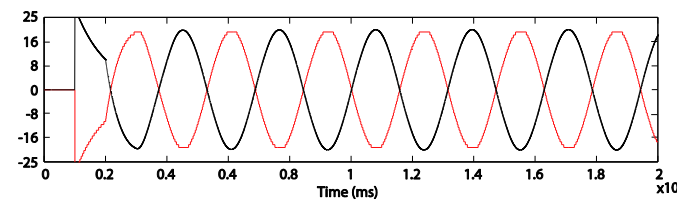
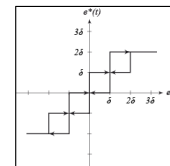
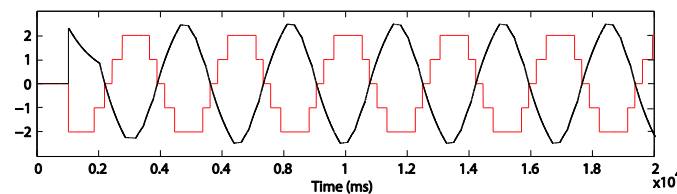
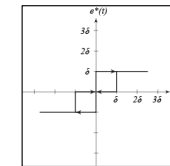
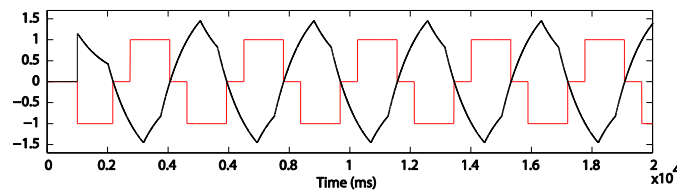
The SSOD sampler allows the system to oscillate in a similar way to that produced by a relay with hysteresis with infinite levels δ (i.e., $\pm \delta, \pm 2 \delta, \dots, \pm n\delta$)!

6. Event based model identification

Identification based on the DF of the SSOD



Different initial conditions produce that the system converges to a **limit cycle** with different amplitude and frequency.



6. Event based model identification

Identification based on the DF of the SSOD

The *describing function* of the SSOD sampler is given by the following equation:

$$N(A, \delta) = \frac{2\delta}{\pi A} \left[1 + \sqrt{1 - \left(\frac{\delta}{A} m\right)^2} + 2 \sum_{k=1}^{m-1} \sqrt{1 - \left(\frac{\delta}{A} k\right)^2} \right] - j \frac{2}{\pi} \left(\frac{\delta}{A}\right)^2 m \quad m = \lfloor A/\delta \rfloor$$

Where the input is a sinusoidal signal of amplitude A .

The condition for the existence of limit cycles is given by:

$$G_{ol}(j\omega_{osc}) = -\frac{1}{N(A, \delta)}$$

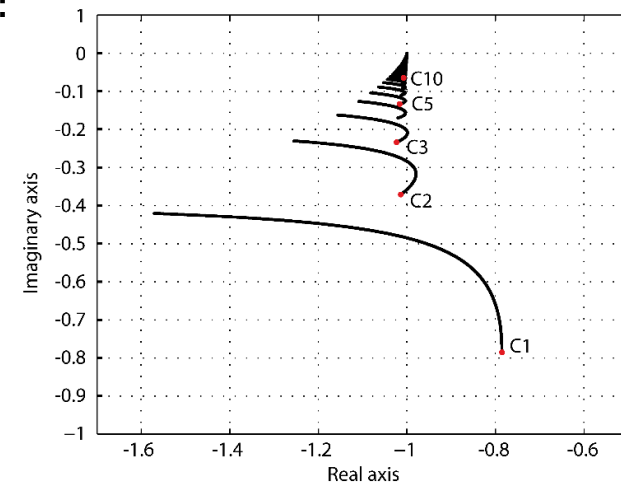
where

$$G_{ol}(j\omega_{osc}) = K_{osc} G(j\omega_{osc})$$

A C_i point corresponds to an amplitude of $A = i\delta$

Dormido, S.; Mellado, M. Determinación de ciclos límite en sistemas de muestreo adaptativo. *Revista de Automática* (1975)

Dormido, S.; Mellado, M. A study on fixed-difference sampling scheme. *Applications and Research in Information Systems and Sciences* (1977)

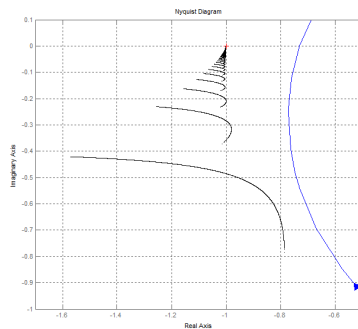


6. Event based model identification

Identification based on the DF of the SSOD

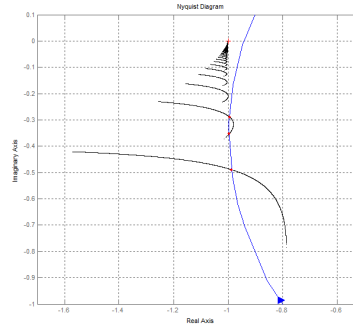
Example of limit cycles by changing K_{osc}

$$G_{ol}(s) = K_{osc} \frac{e^{-s}}{s+1}$$



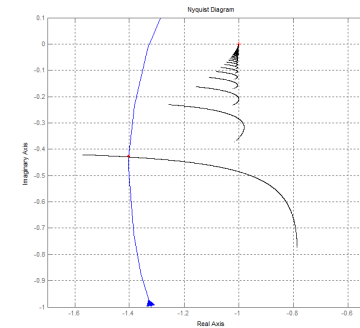
$K_{osc} = 1.65$

Without limit cycle



$K_{osc} = 2.13$

Three possible limit cycle



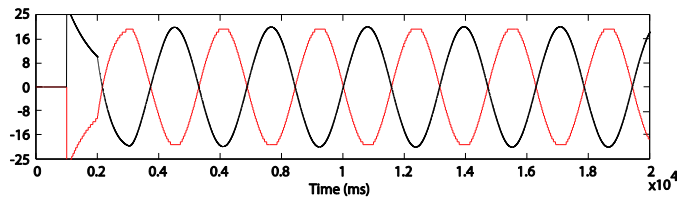
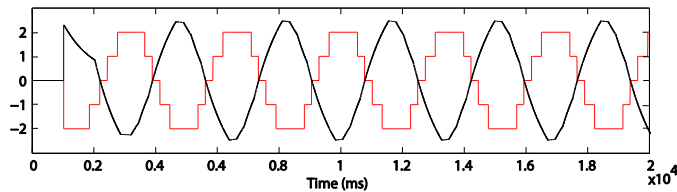
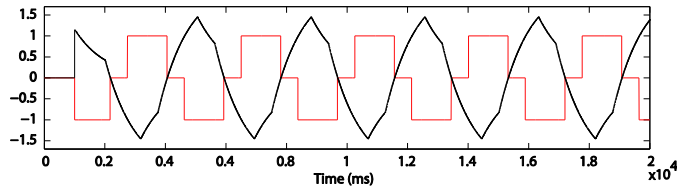
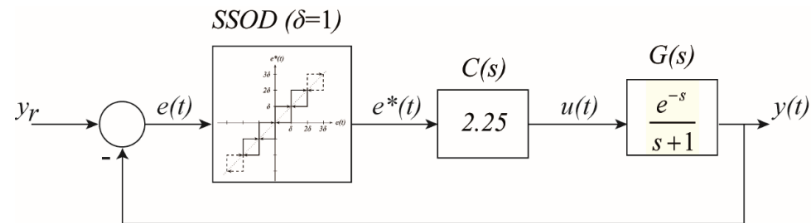
$K_{osc} = 3$

A limit cycle
with possible instability

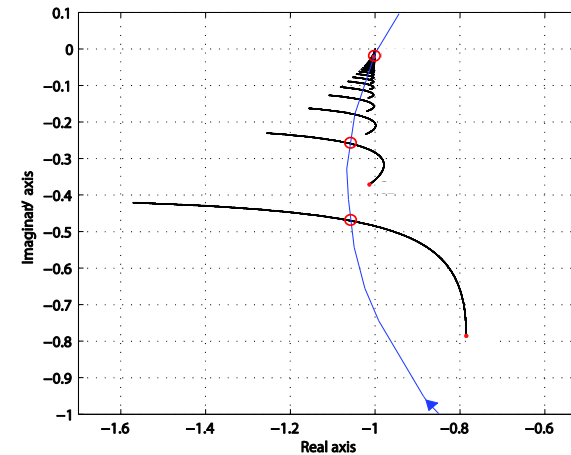
- The theoretical point of intersection C in the plane of Nyquist does not coincide with the real C', but ...
- The greater the amplitude of the oscillation, the information provided by the experimental data matches much more with the information provided by the describing function.
- The greater the amplitude of the oscillation, the better the identification results.

6. Event based model identification

Identification based on the DF of the SSOD



$$\frac{-1}{N(\infty, 1)} = C_\infty = \lim_{i \rightarrow \infty} C_i = -1$$



6. Event based model identification

Identification based on the DF of the SSOD

- Obtain expressions of magnitude and argument of the model to be identified:

$$\begin{array}{l}
 G_{FOPTD}(s) = \frac{Ke^{-Ls}}{Ts+1} \quad G_{SOPTD-1}(s) = \frac{Ke^{-Ls}}{(Ts+1)^2} \\
 G_{FOPDTI}(s) = \frac{Ke^{-Ls}}{s(Ts+1)} \quad G_{SOPTD-2}(s) = \frac{Ke^{-Ls}}{(T_1s+1)(Ts+1)}
 \end{array}
 \quad \Longrightarrow \quad |G(j\omega)| \quad \angle G(j\omega)$$

- We apply a K_{osc} gain to the process so that a stable limit cycle occurs.
- The parameters of the oscillation (ω_{osc} and amplitude A_{osc}) are measured and calculated:

$$C_{osc} = -\frac{1}{N(A_{osc}, \delta)} \quad \Longrightarrow \quad |C_{osc}| \quad \angle C_{osc}$$

- The gain of the process K is obtained adding a *bias* (p.e., 0.1δ) to the output of the SSOD:

$$K = \frac{\int_0^{2\pi/\omega_{osc}} y(t) dt}{K_{osc} \int_0^{2\pi/\omega_{osc}} e^*(t) dt}$$

- The expressions of magnitude are matched to obtain T and those of the argument for L :

$$K_{osc} |G(j\omega_{osc})| = |C_{osc}| \quad \Longrightarrow \quad \angle G(j\omega_{osc}) = \angle C_{osc}$$

6. Event based model identification

Identification based on the DF of the SSOD

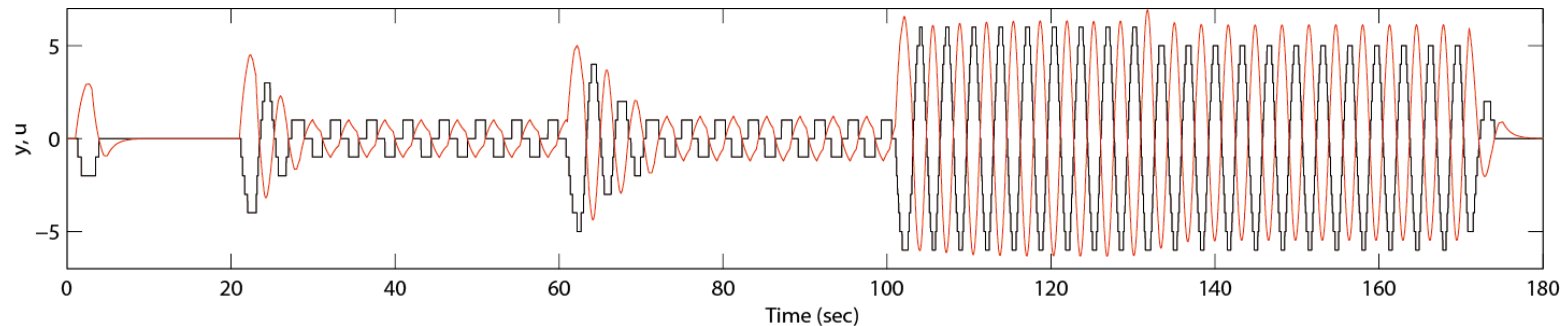
Accuracy of the estimations

Model	Process	C_1	C_2	C_3
$G_{FOPTD}(s) = \frac{Ke^{-Ls}}{Ts+1}$	$\frac{e^{-s}}{s+1}$	$\frac{e^{-1.10s}}{0.78s+1}$ $\omega_{osc}=1.4$ $K_{osc}=1.5$ $A=1.006$	$\frac{e^{-0.99s}}{0.93s+1}$ $\omega_{osc}=1.7$ $K_{osc}=2.0$ $A=2.009$	$\frac{e^{-1.00s}}{0.99s+1}$ $\omega_{osc}=1.843$ $K_{osc}=2.14$ $A=3.02$
$G_{SOPTD-1}(s) = \frac{Ke^{-Ls}}{(Ts+1)^2}$	$\frac{e^{-s}}{(s+1)^2}$	$\frac{e^{-0.99s}}{(0.93s+1)^2}$ $\omega_{osc}=0.96$ $K_{osc}=1.9$ $A=1.01$	$\frac{e^{-1.01s}}{(0.98s+1)^2}$ $\omega_{osc}=1.13$ $K_{osc}=2.3$ $A=2.03$	$\frac{e^{-1.00s}}{(0.99s+1)^2}$ $\omega_{osc}=1.19$ $K_{osc}=2.5$ $A=3.04$

6. Event based model identification

Identification based on the DF of the SSOD

Example of an identification experiment



- K_{osc} increases in jumps of 0.3 until the oscillation is stable

$$\left| \frac{T_{actual} - T_{previo}}{T_{previo}} \right| \leq \varepsilon$$

and of an adequate level (for example, $A_{osc} > 5$).

- A bias of 0.1δ is added in $t=130$ s. to obtain the process gain: $G_{ol}(0) = KK_{osc} = \frac{\int_0^{2\pi/\omega_{osc}} y(t) dt}{\int_0^{2\pi/\omega_{osc}} u(t) dt}$

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7. Conclusions

- Event based control can deal with multi-rate, asynchronism and latency which give difficulties for classical sampled data systems
- Simple examples indicate that event-based control can give good performance, react quickly to disturbances and do nothing when errors are within the tolerance
- Interesting signal form and system structure
- Natural approach for distributed autonomous and multi-agent systems
- Natural for modeling biological systems
- Many interesting open research problems