PID tuning tackling design tradeoffs from an unified perspective

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The universality and resilience of the PID is made explicit with the following sentences from a 2009's article at Machine Design†

- The main benefit of any PID loop is that a designer can "set it and forget it" while still maintaining a well-regulated system.
- PID control is so universal ... PID loops provide technicians and engineers with a customizable way to control a variety of conditions, from temperature to speed and everything in between.

> if PID didn't already exist we would be forced to invent it, or factory automation would be very limited.

†*Paul Avery* Senior Product Training Engineer, Yaskawa Electric, America

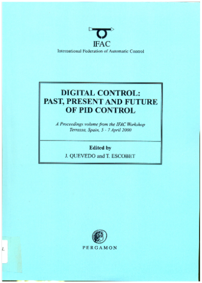
A Survey on Industry Impact and Challenges Thereof

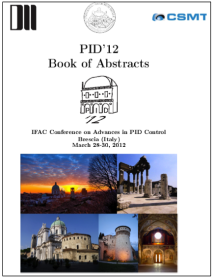
FEBRUARY 2017 « IEEE CONTROL SYSTEMS MAGAZINE 17

Tariq Samad

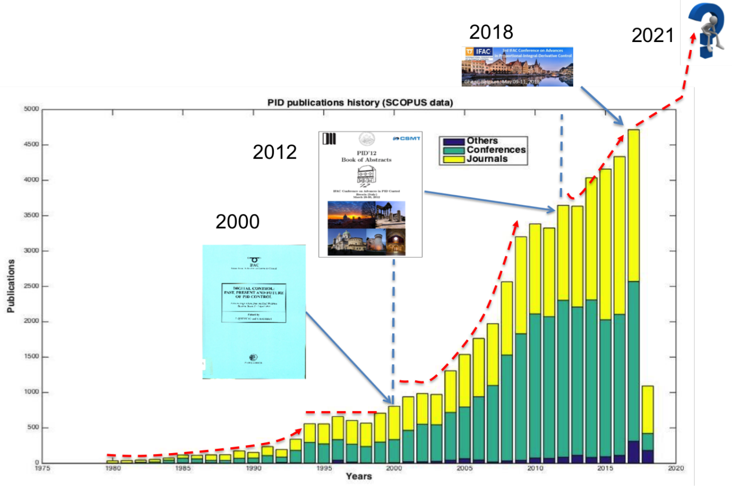
TABLE 1 A list of the survey results in order of industry impact as perceived by the committee members.

Rank and Technology	High-Impact Ratings	Low- or No-Impact Ratings
PID control	100%	0%
Model predictive control	78%	9%
System identification	61%	9%
Process data analytics	61%	17%
Soft sensing	52%	22%
Fault detection and identification	50%	18%
Decentralized and/or coordinated control	48%	30%
Intelligent control	35%	30%
Discrete-event systems	23%	32%
Nonlinear control	22%	35%
Adaptive control	17%	43%
Robust control	13%	43%
Hybrid dynamical systems	13%	43%


IFAC
International Federation of Automatic Control
DIGITAL CONTROL:
PAST, PRESENT AND FUTURE
OF PID CONTROL
A Proceedings volume from the IFAC Workshop
Terrassa, Spain, 5 - 7 April 2000
Edited by
J. QUEVEDO and T. ESCOBET
PERGAMON


CSMT
PID'12
Book of Abstracts
IFAC Conference on Advances in PID Control
Brescia (Italy)
March 28-30, 2012


IFAC
INTERNATIONAL FEDERATION
OF AUTOMATIC CONTROL
3rd IFAC Conference on Advances
in Proportional-Integral-Derivative Control
Ghent, Belgium, May 09-11, 2018


2018
2021
2012
2000
PID publications history (SCOPUS data)
Others
Conferences
Journals
Publications
Years
PID'12
Book of Abstracts
DIGITAL CONTROL: PAST, PRESENT AND FUTURE OF PID CONTROL
IFAC
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The continued interest on PI(D) control is a fact

- From the practitioner's point of view
  - Reliability
  - Smooth control
  - Definition of loop specifications
  - Complement PID with *help units*
  - ...
- From the research point of view
  - Clear guidelines for the benefits of D term
  - Robustness should be transparently included
  - Incorporate tradeoffs and keep simplicity
  - Optimality or suboptimality?
  - ...

What has characterized the evolution of the PID controller and its design approaches is the formulation of tuning rules.

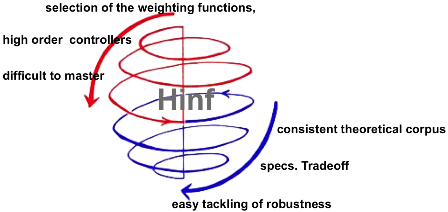
A new PID controller design scenario much more constrained

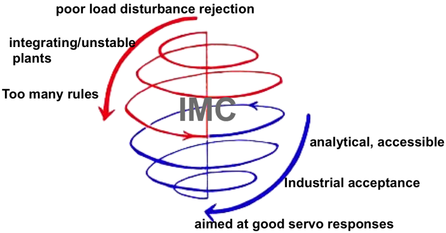
- Servo/Regulation Performance
- Moderate usage of control action
- Robustness
- Simple and clear formulations
- Include derivative term
- Measurement noise attenuation
- *tradeoff* (smoothness/robustness/reactivity)

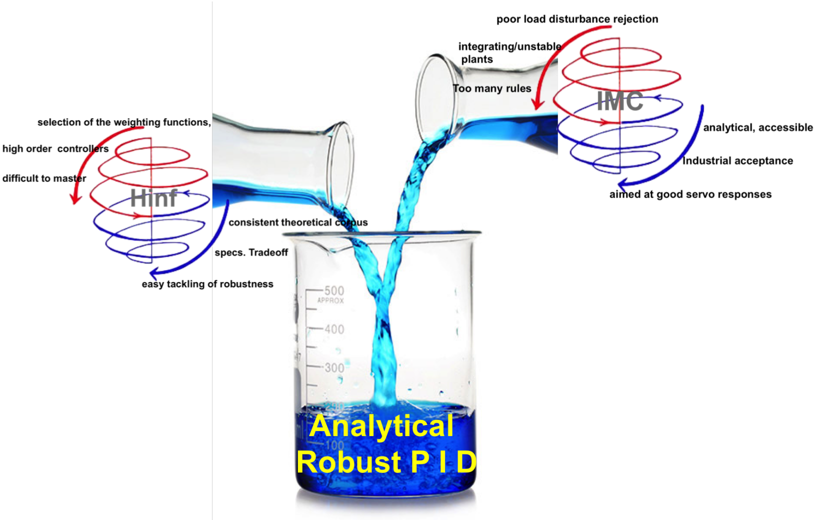
Global perspective

It is needed to rethink how to put play with these considerations, its interactions and formulate the overall design problem in the simplest possible way.

Among modern, advanced, control approaches, there are two well known and succeding approaches:






poor load disturbance rejection
integrating/unstable plants
Too many rules
IMC
analytical, accessible
Industrial acceptance
aimed at good servo responses
selection of the weighting functions,
high order controllers
difficult to master
Hinf
consistent theoretical corpus
specs. Tradeoff
easy tackling of robustness
Analytical
Robust P I D

- State the H_∞ problem for the weighted sensitivity function

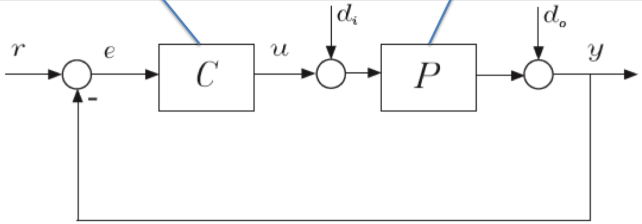
$$\|WS\|_\infty$$

- Appropriately parameterize the weight to represent the problems of interest
  - Servo / Regulation
  - Robustness / Performance
- Analytical solution in terms of the generalized IMC
- Interpret the solution in terms of the PID controller.

Let us consider the standard feedback setup

$$C(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

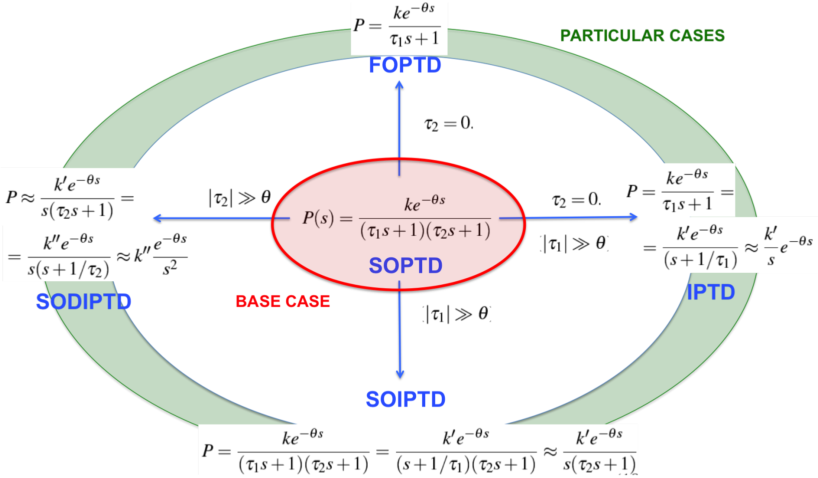
$$P(s) = \frac{k e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$



$$P(s) = \frac{ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

SOPTD

BASE CASE


$P = \frac{ke^{-\theta s}}{\tau_1 s + 1}$
FOPTD
PARTICULAR CASES
$\tau_2 = 0.$
$P \approx \frac{k' e^{-\theta s}}{s(\tau_2 s + 1)} = \frac{k'' e^{-\theta s}}{s(s + 1/\tau_2)} \approx k'' \frac{e^{-\theta s}}{s^2}$
$|\tau_2| \gg \theta$
$P(s) = \frac{ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$
SOPTD
$\tau_2 = 0.$
$|\tau_1| \gg \theta$
$P = \frac{ke^{-\theta s}}{\tau_1 s + 1} = \frac{k' e^{-\theta s}}{(s + 1/\tau_1)} \approx \frac{k'}{s} e^{-\theta s}$
SODIPTD
BASE CASE
$|\tau_1| \gg \theta$
IPTD
SOIPTD
$P = \frac{ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} = \frac{k' e^{-\theta s}}{(s + 1/\tau_1)(\tau_2 s + 1)} \approx \frac{k' e^{-\theta s}}{s(\tau_2 s + 1)}$

For design, we rely on the well-known Weighted Sensitivity problem

$$\min_{C \in \mathcal{C}} \|WS\|_\infty$$

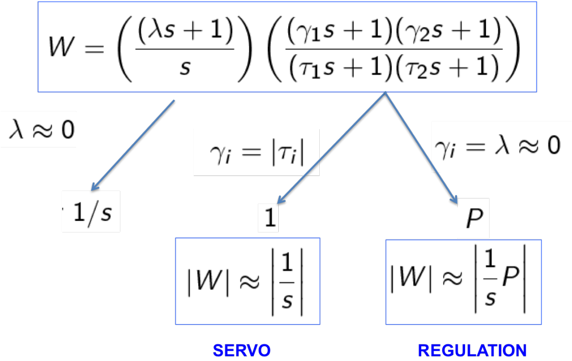
The following structure for the weight is adopted:

$$W = \left(\frac{(\lambda s + 1)}{s} \right) \left(\frac{(\gamma_1 s + 1)(\gamma_2 s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \right)$$

where $\lambda > 0, \gamma_i \in [\lambda, |\tau_i|], i = 1, 2.$
An even more generic weight was analyzed in (Alcantara et al. 2011)[†]

[†] S. Alcántara, W. Zhang, C. Pedret, R. Vilanova, and S. Skogestad, *IMC-like analytical H_∞ design with S/SP mixed sensitivity consideration: Utility in PID tuning guidance*, Journal of Process Control, 2011.

The rationale behind the λ and γ parameters can be explained, quite heuristically, as follows. Remember $\lambda > 0, \gamma_i \in [\lambda, |\tau_i|]$ and start by considering $\lambda \approx 0$, then:



- Intermediate values for γ_1 and γ_2 will produce a balance between the purely servo and regulation situations.

- If we increase the value of λ (we assumed $\lambda \approx 0$), the weight will progressively slow down the resulting closed-loop.

- Once γ_1, γ_2 have been fixed, λ can be used to reach a balance between robustness and performance.

weight selection and tradeoff issues

The selected weight allows us to deal with both tradeoffs

- robustness/performance (via λ)
- servo/regulation issues (via γ_1, γ_2).

The solution of the Weighted Sensitivity problem goes as:

- Parameterize K in the IMC form $K = Q(1 - PQ)^{-1}$
- Rewrite the problem in terms of Q: $\|W(1 - PQ)\|_\infty = \|\mathcal{N}_o\|_\infty$
- The optimal $\mathcal{N}_o$ is all-pass $\mathcal{N}_o = \rho \frac{q(-s)}{q(s)}$
- Recover the optimal $Q_o = P^{-1}(1 - \mathcal{N}_o W^{-1})$
- Get the optimal sensitivity as $S_o = 1 - PQ_o = W\mathcal{N}_o$
- Recover the optimal feedback controller $K_o = Q_o(1 - PQ_o)^{-1}$

In order to analytically solve the weighted sensitivity problem, we first approximate the time delay in the SOPTD model.

$$P = \frac{Ke^{-Ls}}{(\tau_1 s + 1)(\tau_2 + 1)} \approx \frac{k(-\theta s + 1)}{(\tau_1 s + 1)(\tau_2 + 1)}$$

By application of the maximum modulus principle, it turns out that

- the optimal weighted sensitivity is all-pass: $WS^o = \rho$.
- ρ is given by

$$\rho = |W|_{s=\frac{1}{\theta}} = \frac{(\lambda + \theta)(\gamma_1 + \theta)(\gamma_2 + \theta)}{(\tau_1 + \theta)(\tau_2 + \theta)}$$

From the expression for the optimal sensitivity we can get the optimal controller as a PID:

$$C^o = P^{-1}(\rho^{-1}W - 1) = \frac{\zeta_2 s^2 + \zeta_1 s + 1}{\rho k s} = \frac{\zeta_1}{\rho k}\left(1 + \frac{1}{\zeta_1 s} + \frac{\zeta_2}{\zeta_1}s\right)$$

with

$$\zeta_1 = \frac{\theta((\tau_1+\tau_2-\lambda)(\gamma_1+\gamma_2)+\lambda(\tau_1+\tau_2))+\tau_1\tau_2(\gamma_1+\gamma_2+\lambda+\theta)-\gamma_1\gamma_2(\lambda+\theta)+\theta^2(\tau_1+\tau_2)}{(\tau_1+\theta)(\tau_2+\theta)}$$

$$\zeta_2 = \frac{\tau_1\tau_2\big((\lambda+\theta)(\gamma_1+\gamma_2)+\gamma_1\gamma_2+\lambda\theta+\theta^2\big)-\gamma_1\gamma_2\lambda(\theta+\tau_1+\tau_2)}{(\tau_1+\theta)(\tau_2+\theta)}$$

ANALYTIC H_{inf} PID TUNING FOR SOPTD

It is possible however to formulate the analysis that follows with a simplification regarding the $\gamma's$.

- What is strictly necessary in the weight is to have a zero for every plant pole.
- If we have two different $\gamma's$ we will have more freedom for the design, but this is not strictly necessary.
- In order to simplify, we can set $\gamma_1 = \gamma_2 = \gamma$. Then with $\lambda > 0$ and $\gamma \in [\lambda, \tau]$, where we have defined

$$\tau \doteq \max(|\tau_1|, |\tau_2|) = |\tau_1|.$$

$$\min_{C \in \mathcal{C}} \|WS\|_\infty \longleftarrow \boxed{\lambda, \gamma_1, \gamma_2.}$$

$$\downarrow$$

$$\zeta_1 = \frac{\theta\big((\tau_1+\tau_2-\lambda)(\gamma_1+\gamma_2)+\lambda(\tau_1+\tau_2)\big)+\tau_1\tau_2(\gamma_1+\gamma_2+\lambda+\theta)-\gamma_1\gamma_2(\lambda+\theta)+\theta^2(\tau_1+\tau_2)}{(\tau_1+\theta)(\tau_2+\theta)}$$

$$\zeta_2 = \frac{\tau_1\tau_2\big((\lambda+\theta)(\gamma_1+\gamma_2)+\gamma_1\gamma_2+\lambda\theta+\theta^2\big)-\gamma_1\gamma_2\lambda(\theta+\tau_1+\tau_2)}{(\tau_1+\theta)(\tau_2+\theta)}$$

$$\longrightarrow \quad C(s) = \frac{\zeta_2 s^2+\zeta_1 s+1}{\rho k s}$$

$$K_c = \frac{\theta\big(2\gamma(\tau_1+\tau_2-\lambda)+\lambda(\tau_1+\tau_2)\big)+\tau_1\tau_2(2\gamma+\lambda+\theta)-\gamma^2(\lambda+\theta)+\theta^2(\tau_1+\tau_2)}{k(\lambda+\theta)(\gamma+\theta)^2}$$

$$\tau_I = \frac{\theta\big(2\gamma(\tau_1+\tau_2-\lambda)+\lambda(\tau_1+\tau_2)\big)+\tau_1\tau_2(2\gamma+\lambda+\theta)-\gamma^2(\lambda+\theta)+\theta^2(\tau_1+\tau_2)}{(\tau_1+\theta)(\tau_2+\theta)}$$

$$\tau_D = \frac{\tau_1\tau_2\big(2\gamma(\lambda+\theta)+\gamma^2+\lambda\theta+\theta^2\big)-\gamma^2\lambda(\theta+\tau_1+\tau_2)}{\theta\big(2\gamma(\tau_1+\tau_2-\lambda)+\lambda(\tau_1+\tau_2)\big)+\tau_1\tau_2(2\gamma+\lambda+\theta)-\gamma^2(\lambda+\theta)+\theta^2(\tau_1+\tau_2)}$$

$$\longrightarrow \quad C(s) = K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)$$

The previous, messy, expressions constitute the tuning assignment for the most general case. There are some special cases of which deserve specific attention.

- First Order Plus Time Delay (FOPTD)
- Integrating Plus Time Delay (IPTD)
- Second Order Plus Time Delay (SOPTD)
- Second Order Integrating Plus Time Delay (SOIPTD)
- Second Order Double Integrating Plus Time Delay (SODITD)

Will present the corresponding tuning assignments just to show how they derive from the general case.

For a First Order Plus Time Delay (FOPTD) we get a PI controller

$$P(s) = \frac{ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)} \xrightarrow{\tau_2 = 0.} P = \frac{ke^{-\theta s}}{\tau_1 s + 1}$$

$$W = \left(\frac{(\lambda s+1)}{s}\right)\left(\frac{(\gamma_1 s+1)(\gamma_2 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}\right) \xleftarrow{} \gamma = \gamma_1, \gamma_2 = 0$$

$$\boxed{K_c = \frac{\tau_1(\gamma+\lambda+\theta)-\lambda\gamma}{k(\lambda+\theta)(\gamma+\theta)} \quad \tau_I = \frac{\tau_1(\gamma+\lambda+\theta)-\lambda\gamma}{\tau_1+\theta} \quad \lambda > 0, \gamma \in [\lambda, |\tau_1|]}$$

For a Integrating Plus Time Delay (IPTD) we get a PI controller

$$P(s) = \frac{ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)} \xrightarrow[|\tau_1| \gg \theta]{\tau_2 = 0. \quad k' = \frac{k}{\tau_1}} P = \frac{k' e^{-\theta s}}{(s+1/\tau_1)} \approx \frac{k'}{s} e^{-\theta s}$$

$$\downarrow$$

$$W = \left(\frac{(\lambda s+1)}{s}\right)\left(\frac{(\gamma_1 s+1)(\gamma_2 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}\right) \longleftarrow \gamma = \gamma_1, \gamma_2 = 0$$

$$\downarrow$$

$$K_c = \frac{\gamma+\lambda+\theta}{k'(\lambda+\theta)(\gamma+\theta)} \quad \tau_I = \gamma+\lambda+\theta$$

$$\lambda > 0, \gamma \in [\lambda, \infty)$$

For a Second Order Plus Time Delay (SOPTD), this is the original base problem, but we get simple expressions with:

$$\gamma_1 = \gamma_2 = \gamma \quad \tau \doteq \max\{|\tau_1|, |\tau_2|\} = |\tau_1|$$

$$K_c = \frac{\theta(2\gamma(\tau_1+\tau_2-\lambda)+\lambda(\tau_1+\tau_2))+\tau_1\tau_2(2\gamma+\lambda+\theta)-\gamma^2(\lambda+\theta)+\theta^2(\tau_1+\tau_2)}{k(\lambda+\theta)(\gamma+\theta)^2}$$

$$\tau_I = \frac{\theta(2\gamma(\tau_1+\tau_2-\lambda)+\lambda(\tau_1+\tau_2))+\tau_1\tau_2(2\gamma+\lambda+\theta)-\gamma^2(\lambda+\theta)+\theta^2(\tau_1+\tau_2)}{(\tau_1+\theta)(\tau_2+\theta)}$$

$$\tau_D = \frac{\tau_1\tau_2(2\gamma(\lambda+\theta)+\gamma^2+\lambda\theta+\theta^2)-\gamma^2\lambda(\theta+\tau_1+\tau_2)}{\theta(2\gamma(\tau_1+\tau_2-\lambda)+\lambda(\tau_1+\tau_2))+\tau_1\tau_2(2\gamma+\lambda+\theta)-\gamma^2(\lambda+\theta)+\theta^2(\tau_1+\tau_2)}$$

For a Second Order Integrating Plus Time Delay (SOIPTD) we get a PID controller

$$P(s) = \frac{ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)} \xrightarrow{|\tau_1| \gg \theta} P = \frac{k' e^{-\theta s}}{(s+1/\tau_1)(\tau_2 s+1)} \approx \frac{k' e^{-\theta s}}{s(\tau_2 s+1)}$$

$$\gamma = \gamma_1 = \gamma_2$$

$$W = \left(\frac{(\lambda s+1)}{s}\right)\left(\frac{(\gamma_1 s+1)(\gamma_2 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}\right)$$

$$\boxed{\begin{array}{lcl} K_c & = & \frac{(\tau_2+\theta)(\theta+2\gamma+\lambda)}{k'(\lambda+\theta)(\gamma+\theta)^2} \\ \tau_I & = & \theta+2\gamma+\lambda \\ \tau_D & = & \frac{\tau_2\left((\gamma+\theta)^2+\lambda(2\gamma+\theta)\right)-\gamma^2\lambda}{(\tau_2+\theta)(\theta+2\gamma+\lambda)} \end{array} \quad \lambda > 0, \gamma \in [\lambda, \infty)}$$

For a Second Order Double Integrating Plus Time Delay (SODITD) we get a PID controller

$$P(s) = \frac{ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$$

$|\tau_1| \gg \theta$

$k' = \frac{k}{\tau_1}$

$$P = \frac{k' e^{-\theta s}}{(s+1/\tau_1)(\tau_2 s+1)} \approx \frac{k' e^{-\theta s}}{s(\tau_2 s+1)}$$

$|\tau_2| \gg \theta$

$k'' = \frac{k'}{\tau_2}$

$$P \approx \frac{k' e^{-\theta s}}{s(\tau_2 s+1)} = \frac{k'' e^{-\theta s}}{s(s+1/\tau_2)} \approx k'' \frac{e^{-\theta s}}{s^2}$$

$\gamma = \gamma_1 = \gamma_2$

$$W = \left(\frac{(\lambda s+1)}{s}\right)\left(\frac{(\gamma_1 s+1)(\gamma_2 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}\right)$$

$$\begin{aligned} K_c &= \frac{\theta+2\gamma+\lambda}{k''(\lambda+\theta)(\gamma+\theta)^2} \\ \tau_I &= \theta+2\gamma+\lambda \\ \tau_D &= \frac{(\gamma+\theta)^2+\lambda(2\gamma+\theta)}{\theta+2\gamma+\lambda} \end{aligned} \qquad \lambda > 0, \gamma \in [\lambda, \infty)$$

So far, we got a whole set of tuning rules for different process dynamics

- All of them originate from the original weighted sensitivity problem
- The structure of the controller arises from the structure of the problem solution
- All of them are expressed in terms of the $\gamma\lambda$ parameters
- What is left ?
    - Provide selection for $\gamma\lambda$
    - Generate automatic tuning rules

Let us consider the particular example of applying a pure servo and pure regulation design to the process

$$P(s) = \frac{5e^{-s}}{(20s+1)}$$

- Pure servo design: $\lambda = \theta(=1), \quad \gamma = \tau(=20)$

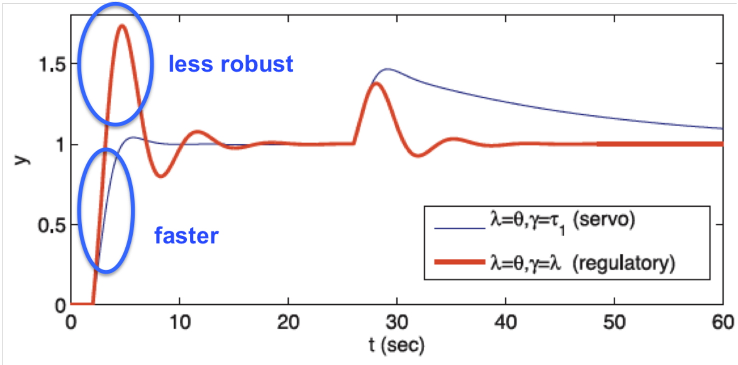
$$Kc = 2.0 \quad T_i = 20.0 \quad M_S \approx 1.6$$

- Pure regulation design: $\lambda = \theta(=1), \quad \gamma = \lambda(=1)$

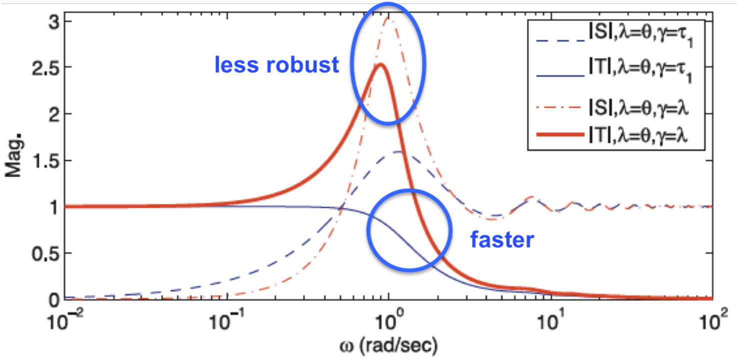
$$Kc = 2.95 \quad T_i = 2.8 \quad M_S \approx 3$$

Regulatory design: faster (large gains) but less robust (higher M_S).

Time domain effects of faster but less robust design



Frequency domain effects of faster but less robust design



The previous situation is not particular of the example but general †:

Regulatory control is based on shifting the slow poles of the plant whereas servo control aims at cancelling them (as long as possible) to flatten out the frequency response.

servos vs. regulation

Therefore, the comparison between the regulator and servo designs is left with a faster and less robust, vs a slower and more robust alternative.

† R. H. Middleton and S. F. Graebe, *Slow stable open-loop poles: to cancel or not to cancel*, Automatica, vol. 35, no. 5, pp. 877-886, 1999.

In (Middleton, 1999)† the notion of Extreme Frequency Equivalence is introduced in order to make two designs comparable:

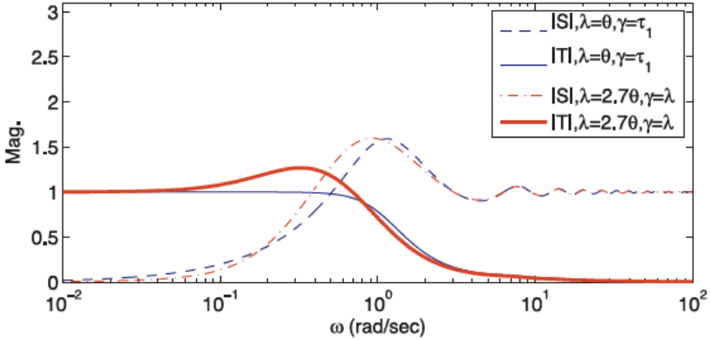
Extreme Frequency Equivalence (EFE)

extreme frequency equivalent complementary sensitivities posses similar initial rise time and the same sensitivity to high-frequency noise and modelling errors.

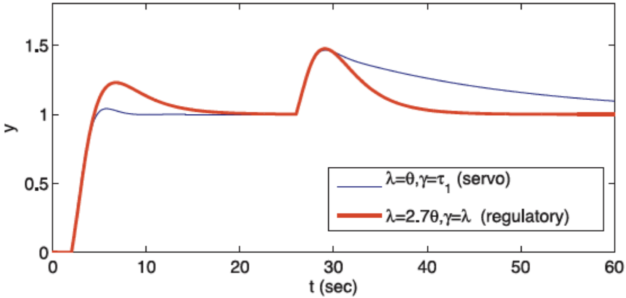
In the previous example we increase the value of λ in the regulator mode until making the servo and regulatory designs comparable in the EFE sense.

† R. H. Middleton and S. F. Graebe, *Slow stable open-loop poles: to cancel or not to cancel*, Automatica, vol. 35, no. 5, pp. 877-886, 1999.

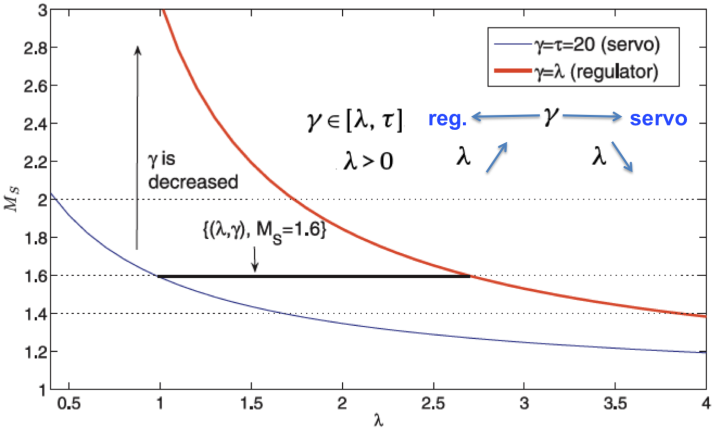
Now we have similar values for the sensitivity peaks (robustness and sensitivity) as well as bandwidth



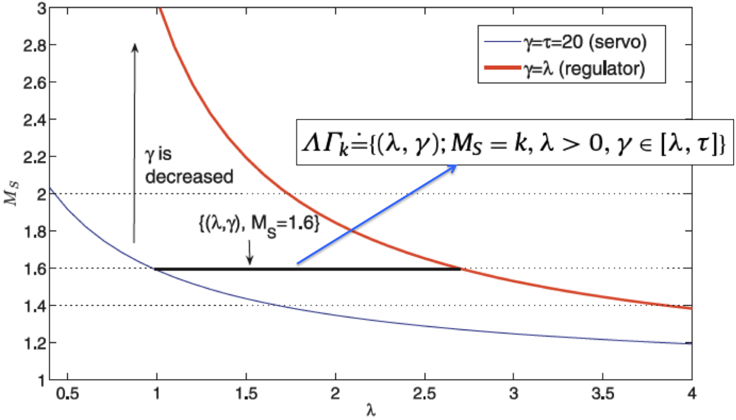
The effect of the EFE can also be appreciated in the time responses.



As we reduce γ to improve regulatory control, then λ has to increase to compensate for robustness (in the EFE sense)



For a specified robustness level we can define the following design space for $\gamma\lambda$



In order to face the problem of balancing the servo and regulatory performance (select γ), we consider the minimization of two alternative performance indices:

$$\begin{aligned} J_{\max} &= \max(\Delta_s, \Delta_r) \\ J_{\text{avg}} &= 0.5(\Delta_s + \Delta_r) \end{aligned}$$

where

$$\Delta_s = \frac{\text{IAE}_s}{\text{IAE}_s^o} \quad \Delta_r = \frac{\text{IAE}_r}{\text{IAE}_r^o}$$

and

$$\text{IAE} = \int_0^\infty |r(t) - y(t)| dt = \int_0^\infty |e(t)| dt$$

The optimal IAE_s^o and IAE_r^o are calculated over $\Lambda\Gamma_k$.

Regarding the two performance indices

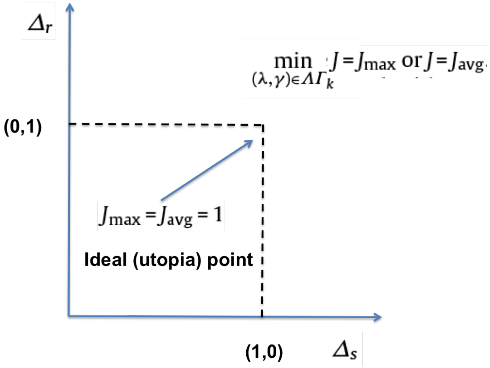
- $J_{\text{avg}} = 0.5(\Delta_s + \Delta_r)$ is used in [1] to evaluate the SIMC-PI method, and it weighs the importance of servo and regulatory performance about equally
- $J_{\max} = \max(\Delta_s, \Delta_r)$ already considered in [2], it adheres to the common strategy in multiobjective optimization of minimizing the worst case performance

Importantly, both are sound performance measures, independent of the process gain, the disturbance and set-point magnitudes, and of the units used for time

[1] C. Grimholt and S. Skogestad, *Optimal PI Control and Verifcation of the SIMC Tuning Rule*, in Proc. of the IFAC Conf. on Advances in PID Control PID'12, 2012.

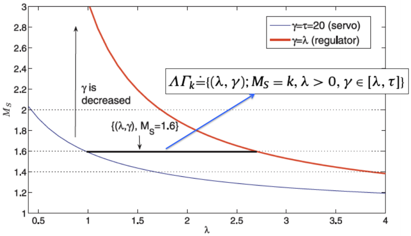
[2] S. Alcántara, R. Vilanova, C. Pedret, and S. Skogestad, *A look into robustness/performance and servo/regulation issues in PI tuning*, in Proc. of the IFAC Conf. on Advances in PID Control PID'12, 2012.

Then, for each robustness level ($M_S = k$), we will consider the following optimization problem



Regarding the definition of the performance degradation and performance indices the following has to be noticed.

- As the design gets more robust, λ increases and the interval for $\gamma \in [\lambda \;\; \tau]$, gets smaller
- In high robustness designs, the servo/regulator trade-off tends to disappear
- $J_{\text{avg}} \approx 1$ and $J_{\max} \approx 1$ are obtained.



This is misleading from a robustness/performance point of view, since high robustness should imply low performance, i.e. $J \gg 1$

In order to study the trade-off between robustness and performance in absolute terms, it will be better to consider the

globally optimal IAE values over the set $\bigcup_k \Lambda\Gamma_k \ \ \forall \ \ k$.

The performance degradation is redefined as follows:

$$\Delta_s^* = \frac{\mathrm{IAE}_s}{\mathrm{IAE}_s^{go}} \ \ , \ \ \Delta_r^* = \frac{\mathrm{IAE}_r}{\mathrm{IAE}_r^{go}}$$

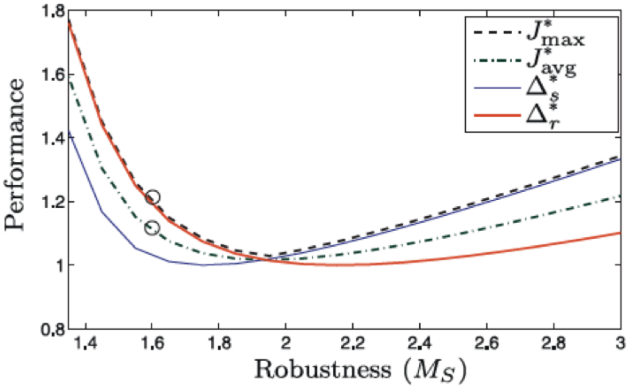
- IAE_s^{go} and IAE_r^{go} are computed over $\bigcup_k \Lambda\Gamma_k$
- IAE_s and IAE_r are computed over $\Lambda\Gamma_k$

Accordingly $J_{\text{avg}}^* = 0.5(\Delta_s^* + \Delta_r^*)$ and $J_{\max}^* = \max(\Delta_s^*, \Delta_r^*)$

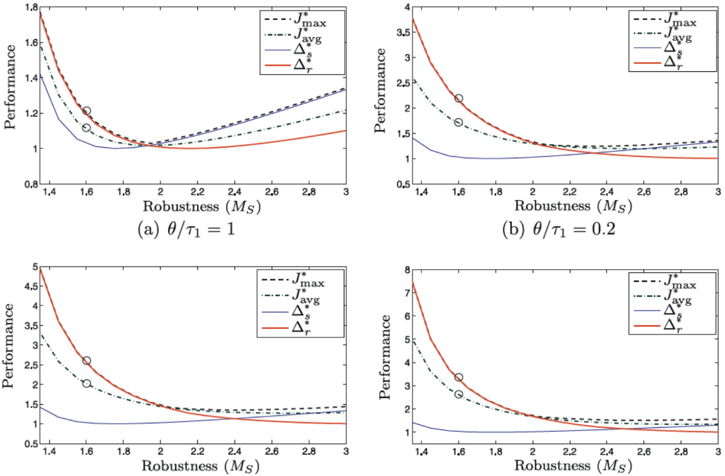
The analysis conducted so far has lead us to generic $\lambda\gamma$ expressions for a PI and a PID.

Now we will proceed to analyze the selection of the tuning parameters λ, γ and to provide tuning guidelines to achieve a *balanced* closed-loop.

1. Analyze the evolution of the performance indexes J^*_{avg} and J^*_{max}
2. Consider the robustness/performance trade-off to see if we can specify a constant target robustness level.
3. Study how to select the λ and γ parameters by solving the optimization problem for $k = M_S^t$.
4. The resulting values for λ and γ are collected
5. Suggest specific choice to generate an automatic tuning.



- Interested in the negative or zero derivative zone
- If $M_s > 2$ both Perf. and Rob can be improved simultaneously
- Servo/regulator tradeoff important is the blue and red plots are separated.
- $M_s^t \approx 1.6$ provides a good choice for the tradeoff



(a) $\theta/\tau_1 = 1$

(b) $\theta/\tau_1 = 0.2$

(c) $\theta/\tau_1 = 0.1$

(d) $\theta/\tau_1 = 0$ ($\tau_1 = \infty$, integrating case)

$\theta/\tau_1 < 1$

$\lambda = 1.25\theta$

$\theta/\tau \geq 1$

$\lambda \approx \theta.$

$\boxed{\lambda = 2.5\theta}$

$J = J_{max}$			$J = J_{avg}$		
θ/τ_1	λ/θ	γ/λ	θ/τ_1	λ/θ	γ/λ
1	1	1	1	1	1
0.2	1.1887	3.1125	0.2	2	1
0.1	1.2406	4.5947	0.1	2.4	1
0.05	1.2718	6.1329	0.05	2.693	1.0026
0.02	1.2840	7.9437	0.02	2.9	1
0 ($\tau_1 = \infty$)	1.3085	9.1708	0 ($\tau_1 = \infty$)	3.0213	1.026

$\boxed{\gamma = \lambda}$

$$P = \frac{ke^{-\theta s}}{\tau_1 s + 1}$$

$$K_c = \frac{\tau_1(\gamma+\lambda+\theta)-\lambda\gamma}{k(\lambda+\theta)(\gamma+\theta)}$$

$$\tau_I = \frac{\tau_1(\gamma+\lambda+\theta)-\lambda\gamma}{\tau_1+\theta}$$

FOPTD

$$K_c = \frac{\gamma+\lambda+\theta}{k'(\lambda+\theta)(\gamma+\theta)}$$

$$\tau_I = \gamma+\lambda+\theta$$

$$P = \frac{k'}{s}e^{-\theta s}$$

IPTD

$\lambda = 2.5\theta \quad \gamma = \lambda$

$J = J_{avg}$

$\theta/\tau_1 < 1$, $\lambda = 1.25\theta$

$\theta/\tau \geq 1$, $\lambda \approx \theta$.

$\lambda = 2.5\theta$

$J = J_{max}$			$J = J_{avg}$		
θ/τ_1	λ/θ	γ/λ	θ/τ_1	λ/θ	γ/λ
1	1	1	1	1	1
0.2	1.1887	3.1125	0.2	2	1
0.1	1.2406	4.5947	0.1	2.4	1
0.05	1.2718	6.1329	0.05	2.693	1.0026
0.02	1.2840	7.9437	0.02	2.9	1
0 ($\tau_1 = \infty$)	1.3085	9.1708	0 ($\tau_1 = \infty$)	3.0213	1.026

$\gamma \approx 9\lambda$

$\gamma = \lambda$

$$\gamma = \min\left(\frac{0.2(\theta/\tau_1) + 0.9}{(\theta/\tau_1) + 0.1}\lambda,\ \tau_1\right)$$

$J = J_{max}$

$\lambda \approx \theta, \quad \theta/\tau \geq 1$

$\lambda = 1.25\theta \quad \theta/\tau_1 < 1$

$$\gamma = \min\left(\frac{0.2(\theta/\tau_1) + 0.9}{(\theta/\tau_1) + 0.1}\lambda,\ \tau_1\right)$$

$\lambda = 1.25\theta$

$\gamma \approx 9\lambda$

$$P = \frac{ke^{-\theta s}}{\tau_1 s + 1}$$

$$K_c = \frac{\tau_1(\gamma + \lambda + \theta) - \lambda\gamma}{k(\lambda + \theta)(\gamma + \theta)}$$

$$\tau_I = \frac{\tau_1(\gamma + \lambda + \theta) - \lambda\gamma}{\tau_1 + \theta}$$

FOPTD

$$K_c = \frac{\gamma + \lambda + \theta}{k'(\lambda + \theta)(\gamma + \theta)}$$

$$\tau_I = \gamma + \lambda + \theta$$

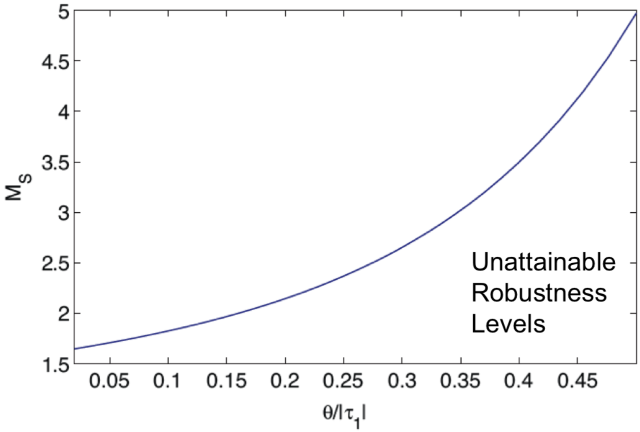
$$P = \frac{k'}{s}e^{-\theta s}$$

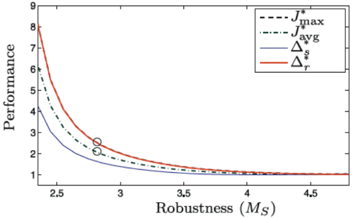
IPTD

$\lambda = 2.5\theta \quad \gamma = \lambda$

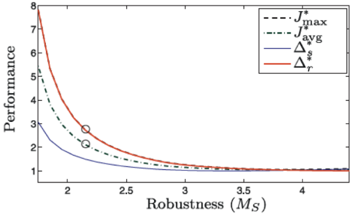
$J = J_{avg}$

For unstable plants M_S gets very large values. One should limit the use of the tuning to plants with $\theta/|\tau_1| < 0.5$ approximately.

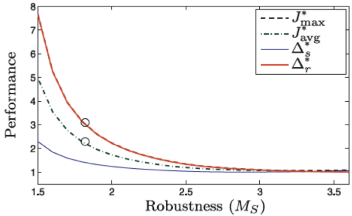




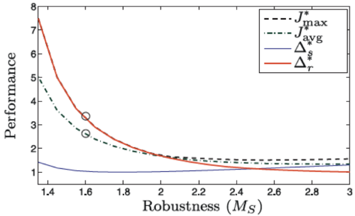
(a) $\theta/\tau_1 = -0.33$



(b) $\theta/\tau_1 = -0.2$



(c) $\theta/\tau_1 = -0.1$



(d) $\theta/\tau_1 = 0$ ($\tau_1 = \infty$, integrating case)

$\lambda = 3\theta;$

		$J = J_{\max}$		$J = J_{\text{avg}}$	
θ/τ_1	k	λ/θ	γ/λ	λ/θ	γ/λ
-0.33	2.9	3	1	3	1
-0.2	2.2	2.9	1	2.9	1
-0.1	1.8	3.1	1	3.1	1
-0.05	1.7	3	1	3	1
-0.02	1.6	3.2	1.03	3.2	1

$\gamma = \lambda$

$$P = \frac{ke^{-\theta s}}{\tau_1 s + 1}$$

$$K_c = \frac{\tau_1(\gamma + \lambda + \theta) - \lambda\gamma}{k(\lambda + \theta)(\gamma + \theta)}$$

$$\tau_I = \frac{\tau_1(\gamma + \lambda + \theta) - \lambda\gamma}{\tau_1 + \theta}$$

UFOPTD

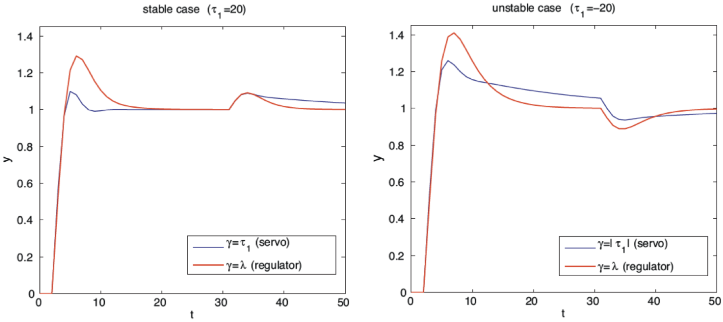
$$\gamma = \lambda \quad \longrightarrow \quad \lambda = 3\theta,$$

$$K_c = \frac{7\tau_1 - 9\theta}{k16\theta}$$

$$\tau_I = \frac{7\tau_1 - 9\theta}{(\tau_1/\theta) + 1}$$

$$J = J_{avg} \quad \& \quad J = J_{max}$$

In this case the analysis conducted suggests regulatory design $(\lambda = \gamma)$ and $\gamma \approx 3\theta$ for both J^*_{avg} and $J^*_{\max}$



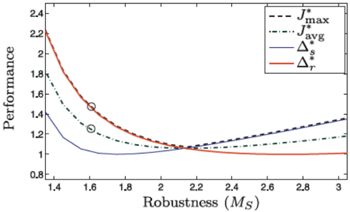
For the second order case we considered the simplification

$$\gamma_1 = \gamma_2 = \gamma \quad \tau \doteq \max\{|\tau_1|, |\tau_2|\} = |\tau_1|$$

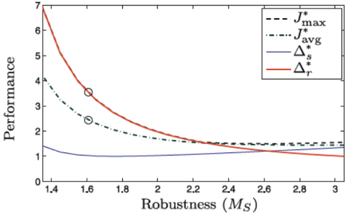
This is well motivated for the double pole case

$$P = \frac{ke^{-\theta s}}{(\tau_1 s + 1)^2}$$

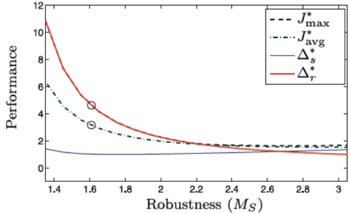
- Analisys gets simple as it only depends on θ/τ
- Results mainly depend on the dominant time constant.
- The resulting tuning guidelines can also be applied to the general SOPTD model for which $\tau_1 \neq \tau_2$



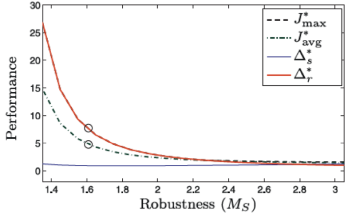
(a) $\theta/\tau_1 = 1$



(b) $\theta/\tau_1 = 0.2$



(c) $\theta/\tau_1 = 0.1$



(d) $\theta/\tau_1 = 0$ ($\tau_1 = \infty$, integrating case)

$\gamma/\lambda = 1.05/((\theta/\tau_1)+0.15)$

$\lambda \approx \theta$ 1.5θ

$J = J_{max}$

θ/τ_1	λ/θ	γ/λ
1	1	1
0.2	1.347	2.8954
0.1	1.4594	4.2483
0.05	1.5755	5.3952
0.02	1.6747	6.4488
$0\ (\tau=\infty)$	1.8542	6.5256

$\lambda = 1.8542\theta$

$\gamma = 6.5256\lambda$

$$\lambda = \left(\frac{0.2(\theta/\tau_1)+1}{(\theta/\tau_1)+0.2}\right)\theta$$

$J = J_{avg}$

θ/τ_1	λ/θ	γ/λ
1	1	1
0.2	2.65	1
0.1	3.3957	1.0307
0.05	4.15	1
0.02	4.72	1
$0\ (\tau=\infty)$	5.2	1

$\gamma = \lambda$

$\gamma = \lambda = 5.2\theta$

$$P = \frac{ke^{-\theta s}}{(\tau_1 s+1)^2}$$

SOPTD

$$K_c = \frac{2\theta(\gamma(2\tau_1-\lambda)+\lambda\tau_1)+\tau_1^2(2\gamma+\lambda+\theta)-\gamma^2(\lambda+\theta)+2\theta^2\tau_1}{k(\lambda+\theta)(\gamma+\theta)^2}$$

$$\tau_I = \frac{2\theta(\gamma(2\tau_1-\lambda)+\lambda\tau_1)+\tau_1^2(2\gamma+\lambda+\theta)-\gamma^2(\lambda+\theta)+2\theta^2\tau_1}{(\tau_1+\theta)^2}$$

$$\tau_D = \frac{\tau_1^2(2\gamma(\lambda+\theta)+\gamma^2+\lambda\theta+\theta^2)-\gamma^2\lambda(\theta+2\tau_1)}{2\theta(\gamma(2\tau_1-\lambda)+\lambda\tau_1)+\tau_1^2(2\gamma+\lambda+\theta)-\gamma^2(\lambda+\theta)+2\theta^2\tau_1}$$

$J = J_{\max}$

$\lambda \approx \theta.\quad 1.5\theta$

$$\gamma = \min\left(\frac{1.05}{(\theta/\tau_1)+0.15}\lambda,\ \tau_1\right)$$

$J = J_{\text{avg}}$

$$\lambda = \left(\frac{0.2(\theta/\tau_1)+1}{(\theta/\tau_1)+0.2}\right)\theta$$

$\gamma = \lambda$

$$P \approx \frac{k' e^{-\theta s}}{s(\tau_2 s + 1)} = \frac{k'' e^{-\theta s}}{s(s + 1/\tau_2)} \approx k'' \frac{e^{-\theta s}}{s^2}$$

$$K_c = \frac{\theta + 2\gamma + \lambda}{k''(\lambda + \theta)(\gamma + \theta)^2}$$

$$\tau_I = \theta + 2\gamma + \lambda$$

$$\tau_D = \frac{(\gamma + \theta)^2 + \lambda(2\gamma + \theta)}{\theta + 2\gamma + \lambda}$$

SODIPTD

$J = J_{max}$

$\lambda = 1.8542\theta$

$\gamma = 6.5256\lambda$

$$K_c = \frac{0.055}{k''\theta 2}$$

$$\tau_I = 27\theta$$

$$\tau_D = 8\theta$$

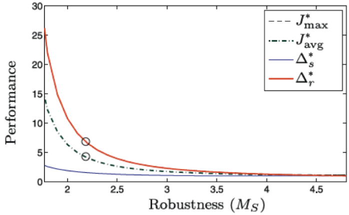
$J = J_{avg}$

$\gamma = \lambda = 5.2\theta$

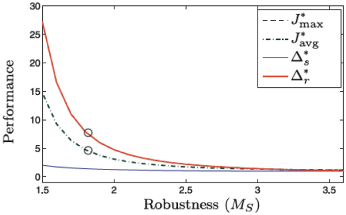
$$K_c = \frac{0.07}{k''\theta 2}$$

$$\tau_I = 16.6\theta$$

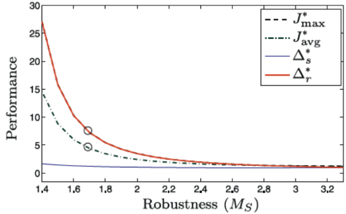
$$\tau_D = 5.9\theta$$



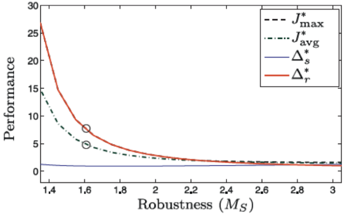
(a) $\theta/\tau_1 = -0.1$



(b) $\theta/\tau_1 = -0.05$



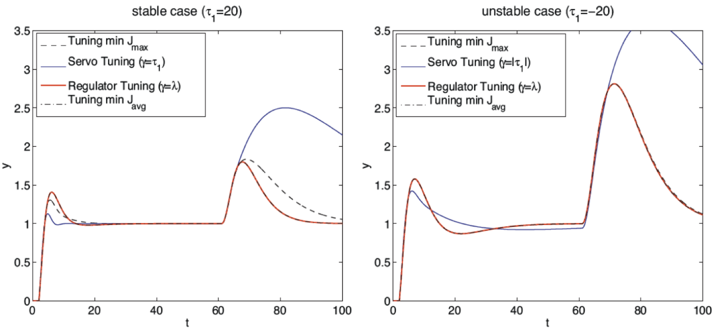
(c) $\theta/\tau_1 = -0.02$



(d) $\theta/\tau_1 = 0$ ($\tau_1 = \infty$, integrating case)

In this case the analysis conducted suggests regulatory design ($\lambda = \gamma$) and $\gamma \approx 5.2\theta$ for both J^*_{avg} and J^*_{max}

$P = 50e^{-s}/(\tau_1 s + 1)^2 \quad M_S = 1.8.$



- It is possible to encompass PI/D design within a modern control theory viewpoint
- The problem is solved in the two parameter space $\lambda\gamma$ instead of the original K_c, T_i, T_d space
  - we may not get the truly optimal
  - better to analyze the robustness/performance - servo/regulation tradeoff
- The treatment is unifying as the same approach and root solution applies for a ser of process dynamics
- for particular selections, tuning result into already know ones
  - $\gamma = \tau_1$ we get the IMC designs
  - other particular cases for concrete published tunings

Regarding the servo/regulation performance, two indexes have been considered: J^*_{avg} and J^*_{max}

- For stable dynamics J^*_{avg} favours regulatory operation
- For stable dynamics J^*_{max} results in a more balanced servo/regulator trade-off
  - sometimes yielding a sluggish closed-loop
  - designs less sensitive to modelling errors
- For unstable dynamics the option is to go for regulatory operation

Confirms usual practice

Regulatory control is the preferred option in general terms, justifying the common practice of considering only input disturbances, There is still much work to be done:

- For FO dynamics PI is suggested,... but would PID provide other benefits?
- What about oscillating dynamics
- Explore different ways of wheigthing servo/regulation/robustness
- Implications of other (more practical) formulations for the PID

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PID control in terms of *robustness/performance* and *servo/regulator* trade-offs: A unifying approach to *balanced* autotuning

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Generalized Internal Model Control for Balancing Input/Output Disturbance Response

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ABSTRACT: Based on internal model control (IMC), we present a design method to take into account both input and output disturbances. The proposed design provides generalized IMC filters that can be used to obtain good results in terms of output sensitivity (favoring output disturbances), or in terms of input sensitivity (therefore placing the emphasis on load disturbances). If both input and output disturbances are expected, the design offers the possibility of obtaining a balance that improves the overall disturbance rejection response.

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IMC-like analytical $\mathcal{H}_\infty$ design with S/SP mixed sensitivity consideration: Utility in PID tuning guidance[☆]

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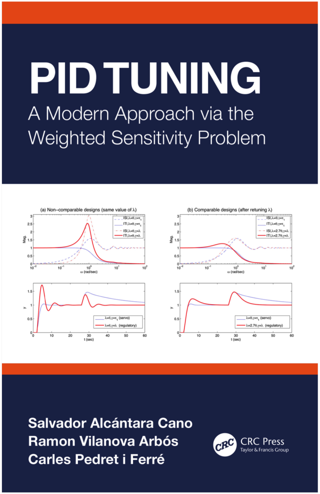

PID tuning tackling design tradeoffs from an unified perspective

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Universitat Autónoma de Barcelona, Barcelona

Workshop *"Advanced Topics in PID Control System Design, Automatic Tuning and Applications"*
21st IFAC World Congress, Germany
July 12, 2020